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PISA FUNCTIONAL LITERACY

AS REPRESENTED IN TAIWANESE MATHEMATICS TEXTBOOKS

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PISA is the largest international educational assessment coordinated by the Organization for Economic Co-operation and Development (OECD). PISA's *"Functional Literacy"* emphasizes the theoretical concept of mathematics as a human activity. From this pedagogical point of arrival, the *"The mathematization cycle"* serves as an instructional cycle which is a crucial feature in PISA's designed assessment. PISA's *"Functional Literacy"* stresses the significant of mathematics education reflects on applied, authentic and heuristic surrounded by individual's daily life in society. In addition, PISA's assessments aim to assess students' mathematics competency level in a comprehensible way - the *cognitive demands.* PISA utilized three competence clusters – *reproduction*, *connections*, and *reflections* as well as the six levels of literacy scale to assess the *cognitive loads.*

Taiwan is a developing democratic country. Taiwanese students have performed advanced ranking in international mathematics assessments – PISA and TIMSS. Taiwan's mathematics textbooks determine the thinking and direction of education. Could it possibly due to characteristics of Taiwan's mathematics textbooks? This study aims to demonstrate the "*Functional Literacy*" as presented in Taiwanese mathematics textbooks. This study was focused on the content and design of Taiwanese mathematics textbook problems for secondary school. The main interest was to explore the designed *mathematization cycle* methods in the mathematics textbooks' problems and to measure the level of the *cognitive loads* in which student's solution required to use in accordance with PISA's description. The results of the analyses in this study show that selected instructional cycles in Taiwanese mathematics textbooks do demonstrate an ultimate match with PISA's *"The mathematization cycle"* and high level of *cognitive demands.*

The finding revealed that PISA's theoretical concept and how it can be implemented for the development of mathematics textbooks. Consequently, the results of different methods may create a new trend in the process of instructional curriculum in mathematics education. These

theoretical findings are reflected in international assessment results. The process in this study can provide an example of the basic mathematical concept and their applications could advance in assessment outcome. Suggestions are given to teachers and further studies in textbook and curriculum for improving the mathematics education.

Keywords: PISA, outcome, mathematics textbook, functional literacy, instructional, mathematization cycle, cognitive demands, apply, authentic, reproduction, connections, reflections, mathematical competencies

Introduction

The Program for International Student Assessment (PISA) is a worldwide, internationally standardized assessment program developed jointly by participating countries. It is the largest international educational assessment activity ever conducted. Coordinated by the Organization for Economic Co-operation and Development (OECD), PISA is intended to improve educational policies and outcomes. The PISA assessment was first conducted in 2000 and is repeated every three years. Typically, tests are administered to between 4,500 and 10,000 students in each country. In 2006, approximately 400,800 students worldwide participated in the assessment, which by that time had represented 32 million 15-years-olds. Taiwan (Republic of China) participated and ranked "first place" among 57 countries. By 2009, 74 countries had taken part in the assessment, and China (People's Republic of China), which was participating for the first time, achieved first place in the rankings.

Mathematical functional literacy incorporated within PISA deals with the capacity of students (at the end of their compulsory schooling) "to analyze, reason and communicate efficiently as they pose, formulate, solve and interpret mathematical problems in a variety of situations" (OECD/PISA, 2003, p. 24). PISA claims to measure applications to real-life problems and life-long learning (workforce knowledge).

PISA's mathematics assessment design is based on the following literacy perspective:

> Mathematical literacy is an individual's capacity to identify and understand the role that mathematics plays in the world, to make well-founded judgments and to use and engage with mathematics in ways that meet the needs of that individual's life as a constructive, concerned and reflective citizen. (OECD/PISA 2003 Assessment Framework)

PISA focuses on the extent to which students can use the knowledge and skills they have learned and practiced at school when confronted by situations and challenges for which that knowledge may be relevant. In sum, PISA aims to assess schooling outcomes in terms of whether students can put mathematics to functional use in their lives.

The aim of the OECD/PISA assessment is to develop indicators of the extent to which the educational systems in participating countries have prepared 15-year-olds to play constructive roles as citizens in society. Rather than being limited to the curriculum content students have learned, the assessment focus is on determining if students can use what they have learned in situations they are likely to encounter in their daily lives (OECD/PISA, 2003, p. 24).

The OECD/PISA's definition of mathematical functional literacy is consistent with the broad and integrative theory about the structure and use of mathematics, as reflected in recent socio-cultural literacy studies. A person who is literate in mathematics would know many of the design resources of mathematics and could use them for different social *functions*. That person must also learn to use such ideas to solve non-routing problems in various situations (OECD/PISA, 2003, p. 26). The Organization for Economic Co-Operation and Development (OECD) (2002) claims that teaching students to "mathematize" should be a primary goal of mathematics education. PISA was designed and developed by the OECD in the late 1990s as an ongoing, periodic, international comparative study of certain student characteristics and proficiencies. It was designed to generate indicators of aspects of educational performance using high-quality and reliable measures. PISA is managed and directed cooperatively by OECD member countries in cooperation with a large and increasing number of non-member countries referred to as "partner countries." The OECD administers the project through a small Secretariat based in Paris. PISA's surveys take place every three years. The first survey was conducted in 2000. PISA's assessment focuses on young people's ability to use their knowledge and skills to meet real-life challenges, rather than merely determining the extent to which they have mastered a specific school curriculum. This approach is called "functional literacy". That is, it assesses the extent to which students can use their reading skills to understand and interpret various kinds of written material that they are likely to meet as they negotiate their daily lives; the extent to which students can use their mathematical knowledge and skills to solve various kinds of mathematics-related challenges and problems they are likely to meet and the extent to which students can use their scientific knowledge and skills to understand, interpret and resolve various kinds of scientific situations and challenges. (Take the Test, Sample Questions from OECD's PISA Assessments, 2009, p. 13)

Mathematical literacy cannot be reduced to, but certainly presupposes, knowledge of mathematical terminology, facts, and procedures as well as skills in performing specific operations and carrying out certain methods. Mathematical literacy involves the creative combining of these elements in response to the demands imposed by external situations such as events in the world and in life, and how an individual uses and engages with them (Take the Test, Sample Questions from OECD's PISA Assessments, 2009, p. 13). The origins of the OECD itself date back to 1960, when 18 European countries plus the United States and Canada joined forces to create an organization dedicated to global development. Today, OECD's 34 member countries span the globe, from North and South America to Europe and the Asia-Pacific region. Countries that are not members are considered "partner" countries. Currently, 25 non-members participate as regular observers or full participants in OECD Committees. About 50 Non-members are engaged in OECD working parties, schemes or programs. Among PISA's participating "Partner Countries," Taiwan, Hong Kong, China, and Estonia were identified as the four best-performing. The purpose of this study was to demonstrate the extent to which Taiwanese mathematics textbooks are compatible with the PISA concept of mathematical functional literacy in junior school education (ages 13-15) and senior school education (ages 16-18). The following two research questions guided this study:

1. In what ways do Taiwanese mathematics textbooks demonstrate support of the PISA concept of mathematical functional literacy?
2. What level of cognitive demands is present in Taiwanese mathematics textbooks in comparison to PISA's cognitive demands?

Taiwan's Mathematics Education

Taiwan was ranked first in eighth grade mathematics in the 2007 TIMSS assessment. Kao suggests that Taiwan is advanced in implementing the conception of PISA's mathematics literacy (Kao, 2009), possibly due to the characteristics of that country's mathematics textbooks. The textbook is an integral part of the educational process and has a direct impact on that process. It is the apparatus of control not only for pupils' learning but also for teachers' instruction. During the teaching and learning process, a textbook performs several functions for its users, whether students or teachers (Břehovský & Emanovský, 2011, p.2). Garner (1992) notes that "textbooks serve as critical vehicles for knowledge acquisition in school" and can "replace teacher talk as the primary source of information" (p. 53).

Taiwan's textbooks are published by private publishing companies; however, the Ministry of Education approves the textbooks based on whether they meet national standards. Four Taiwanese eighth and ninth grade primary mathematics textbooks were examined. Two textbooks (identified as #1 and #2) were from the third of three years of junior school education; two textbooks (identified as #3 and #4) were from the first of three years of senior school education.

Recent Amendments to Taiwan's High School Mathematics Syllabus

In 1997, the government passed an amendment to the regular high school mathematics syllabus, which emphasized the important aspects of consistency, articulation, connectivity, appropriateness, and international scope. The effect of the amendment is to combine various disciplines that conduct mathematics in the future; therefore, the future development of mathematics for students will become increasingly important. According to UNESCO, the trend of developing various disciplines of knowledge flow will be mathematical. It is important that learning philosophies follow the foundation of lifelong learning. The important purpose of students learning mathematics in high school is to lay a solid foundation of coherence for development of individuals and society as a whole. The core content for the designed syllabus is defined by *daily needs* or *the needs of other disciplines*. Mathematics learning should focus on training mathematical thinking that is "stacking lots of problem-solving skills and "delivering a culture of learning"; thus, the method to develop solid mathematical problem-solving skills is an important part of training. Designed questions on various situations need to be related to ordinary management in the way that they should all be artificially tricky. Five learning goal were implemented: *Consistency, Articulation, Connectivity, Appropriateness,* and *International.*

OECD/PISA's Criteria for Mathematics Functional Literacy

This investigator investigates and demonstrates in this study by showing how OECD/PISA designs their tests and what their theoretical basis is. The OECD/PISA design features for mathematics literacy include knowing the basic terms, procedures, and concepts commonly taught in schools and knowing how these features are structured and used. These scholarly notions involving the interplay of "design features" and "functions" that support the

OECD/PISA mathematics framework are illustrated in "*The Mathematization Cycle*" and their characteristics—applied, authentic, and heuristic—are necessary for functionality (OECD/PISA, 2003, p. 26). The strategy for this framework is combining the theoretical basis with these characteristics to unify the functional aspect of literacy. Ensuring cultural appropriateness involved using a wide variety of materials that representing different cultural experiences and contexts. Then, those materials were processed and refined to ascertain that different interests were well-balanced and that the selected materials would work well in all countries. Materials were revised on the basis of field trial data analyses, and the final selection of material for the main PISA study was guided by including *only fully functioning test items across all national and language versions.* At the conclusion of this process, each national version produced can be regarded as linguistically and psychometrically equivalent to the source versions, and therefore capable of contributing to the estimation of a single international set of item parameters (Turner, 2007).

Forman and Lynn (2000) indicate that "functional mathematics provides a rich foundation of experience and examples on which students can build subsequent abstractions and generalizations" (p. 11). Since the goal of the OECD/PISA is to access students' capacity to solve real problems, its strategy is to describe the range of content in relation to the phenomena and the kinds of problems for which it was created. This approach ensures a focus on the assessment that is consistent with the domain's definition, yet covers a range of content that includes what is typically found in other mathematical assessments and in national mathematics curricula (OECD/PISA, 2003, p. 34).

Demonstrate how to answer the research questions

In the OECD/PISA, the fundamental process by which students solve a real-life problem is referred to as "mathematization." OECD/PISA's mathematization cycle parallels Freudenthal's Realistic Mathematics Education. OECD/PISA transforms RME into the infrastructure of an instructional process and the theoretical basis for functional literacy in the mathematics curriculum. OECD/PISA defines the steps of the mathematization cycle as follows:

To answer the first research question, five instructional cycles[1] in each of four Taiwanese textbook examples were identified that:

1. began with a problem situated in reality;
2. organized mathematical concepts and identified the relevant mathematics;
3. transformed the real-world problem into a mathematical problem that faithfully represents the situation;
4. solved the mathematical problem; and
5. made sense of the mathematical solution in terms of the real situation, including identifying the limitations of the solution.

The five instructional cycles were examined to determine whether each selected example was consistent with PISA's philosophical description of functional literacy.

The investigator chose two methods from PISA to answer the second research question. 1) PISA scaled the exam questions on six literacy levels, each level requiring the students to achieve different levels of cognitive activities as defined by PISA. 2) According to PISA, the mathematical processes that students apply as they attempt to solve problems are referred to as *mathematical competencies.* PISA defined three competency clusters that encapsulate the

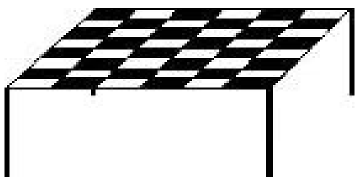
[1]OECD/PISA's "*The mathematization cycle*" in this study is referred to as an instructional cycle.

different cognitive processes needed to solve various kinds of problems. These three clusters reflect the way mathematical processes are typically employed when solving problems that arise as students interact with their world (p. 37). To answer the second research question, the difficulty levels of the *cognitive demands* of the five instructional cycles were examined to determine whether PISA's descriptions were followed. One way to accomplish this in a comprehensive way was to describe the three cluster competencies—*reproduction*, *connections*, and *reflections*—that were designed by PISA as well as the six levels of the literacy scale that PISA utilizes in its assessment. Thus, *mathematical competencies* are one indicator that contributes to the cognitive load in Taiwan's mathematics textbooks. Therefore, to answer research question two, the first step was to evaluate the literacy difficulty level following PISA's description, then analyze the competency cluster for the selected instructional cycle individually. The combination of the results from the two methods—literacy difficulty level and competency cluster—provides evidence of the cognitive load presented in Taiwan's textbooks in comparison to PISA's cognitive demands.

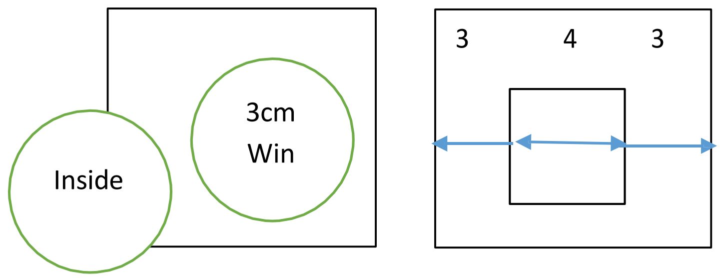
The OECD/PISA assessed mathematical literacy in 2003 through a combination of items with open constructed-response types, closed constructed-response types, and multiple-choice types. Open constructed-response items require more extended responses from students, and the process of producing a response frequently involves higher-order cognitive activities. Based on experience in developing and using test items for OECD/PISA 2000, the multiple-choice type is generally regarded as most suitable for assessing items that are associated with the Reproduction and Connection competency clusters with a limited number of defined response-options. For response options, students must translate the problem into mathematical terms, devise a model to represent the periodic nature of the context described, and extend the pattern to match the result with one of the given options (OECD/PISA, 2003. pp. 50-52).

Figure 1. A PISA example (Source: OECD/PISA, 2003, pp. 28, 29)

Fairground Game board: at a fair, players throw coins onto a board checkered with squares. If a coin touches a boundary, it is lost. If it is rolls off the board, it is returned. But if the coin lies wholly within a square, the player wins the coin back plus a prize. What is the probability of winning at this game? The square is 10 x10cm.
Inside the circle wins Winning toss and losing toss and the sample and event spaces



Inside the circle wins



Winning toss and losing toss and the sample and event spaces

1) Begin with a problem situated in reality; Clearly this exercise is situated in reality.

2) Organize mathematical concepts and identify the relevant mathematics;

First, the students began by realizing that the probability of winning depends on the relative sizes of the squares and the coin (identifying the important variables).

3) Transforms the real-world problem in to a mathematical problem that faithfully represents the situation;

4) Next, to transform the real problem into a mathematical problem, they realized that it might be better to examine the relationship for a single square and a smaller circle (trimming the reality).Then they decided to construct a specific example (using a problem solving *heuristic*). Students let the radius of the coin be 3cm and the side of the squares is 10cm. They realized that to win, the centre of the coin must be at least 3cm from each side; otherwise the edge of the coin will fall across the square. The sample space was the square with side 10cm, and the winning event space was a square with side 4cm. The relationships are shown in the following diagram (Figure 2).

5) Solve the mathematical problem; and
Make sense of the mathematical solution in terms of the real situation; including identification of the limitations of the solution.
The probability of winning was obtained from the ratio of the area of the sample and event space squares (for example p = 16/100). Then the students examined coins of other sizes, and generalized the problem by expressing its solution in algebraic terms. Finally the students extended this finding to work out the relative sizes of the coin and squares for a variety of practical situations; they constructed boards and empirically tested results (making sense of the

Summary Descriptions for Six Levels of Overall Mathematical Literacy

At Level VI students can conceptualize, generalize, and utilize information based on their investigations and modeling of complex problem situations. They can link different information sources and representations and flexibly translate among them. Students at this level are capable of advanced mathematical thinking and reasoning. These students can apply this insight and understandings along with a mastery of symbolic and formal mathematical operations and relationships to develop new approaches and strategies for attacking novel situations. Student at this level can formulate and precisely communicate their actions and reflections regarding their findings, interpretations, arguments, and the appropriateness of these to the original situations.
At Level V students can develop and work with models for complex situations, identifying constraints and specifying assumptions. They can select, compare, and evaluate appropriate problem solving strategies for dealing with complex problems related to these models. Students at this level can work strategically using broad, well-developed thinking and reasoning skills, appropriate linked representations, symbolic and formal characterizations, and insight pertaining to these situations. They can reflect on their actions and formulate and communicate their interpretations and reasoning.
At Level IV students can work effectively with explicit models for complex concrete situations that may involve constraints or call for making assumptions. They can select and integrate different representations, including the symbolic, linking them directly to aspects of real-world situations. Students at this level can utilize well-developed skills and reason flexibly, with some insight, in these contexts. They can construct and communicate explanations and arguments based on their interpretations, arguments, and actions.
At Level III students can execute clearly described procedures, including those that require sequential decisions. They can select and apply simple problem solving strategies. Students at this level can interpret and use representations based on different information sources and reason directly from them. They can develop short communications reporting their interpretations, results and reasoning.
At Level II students can interpret and recognize situations in contexts that require no more than direct inference. They can extract relevant information from a single source and make use of a single representational mode. Students at this level can employ basic algorithms, formulae, procedures, or conventions. They are capable of direct reasoning and making literal interpretations of the results.
At Level I students can answer questions involving familiar contexts where all relevant information is present and the questions are clearly defined. They are able to identify information and to carry out routine procedures according to direct instructions in explicit situations. They can perform actions that are obvious and follow immediately from the given stimuli.

The most important goal of cognitive demand for the OECD/PISA is to provide a variety of mathematical problems with varying degrees of built-in guidance and structure, but pushing towards *authentic* problems for which students must do the thinking themselves (OECD/PISA, 2003, p. 56).The OECD/PISA describes the cognitive activities that these competencies encompass according to three “competency clusters”: the *Reproduction* Cluster, the *Connection* Cluster, and the *Reflection* cluster (see Figure 2).

Figure 2. Diagrammatic representation of the competency clusters

Mathematical Literacy

The Reproduction Cluster	**The Connection Cluster**	**The Reflection Cluster**
Standard representations and definitions	Modeling	Complex problem-solving and posting
Routing Computations	Standard problem-solving translation and interpretation	Reflecting and insight
Routine procedures	Multiple well-defined methods	Original mathematical approach
Routing problem-solving		Multiple complex methods
		Generalization

Demonstrate PISA’s Functional Literacy as Presented in

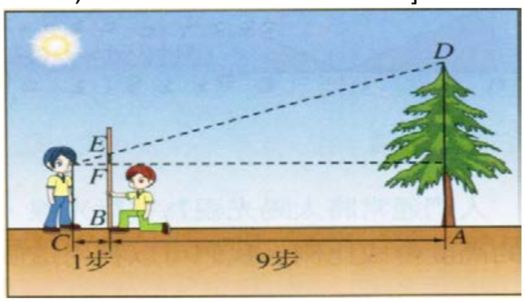
Taiwanese Mathematics Textbooks

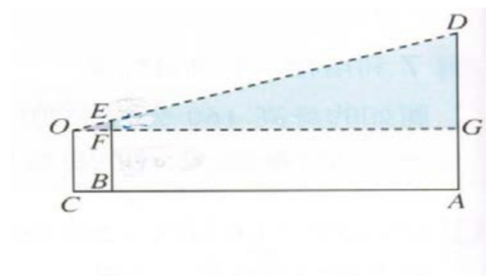
In this version of research paper will show four instructional cycle examples verses 13 examples. Selected instructional cycles from each chapter , each textbook were determined to best represent the mathematics lesson cycle and demonstrate PISA’s functional literacy. Each cycle involves the interplay of “design features” and “functions” that support the mathematics framework for OECD/PISA.

Instructional Cycle 1

Book #1 Chapter 1, Example 8, p.50: utilize the similar figures to estimate the high of a tree

Little Ming and Big Quoi want to measure the high of a tree. Little Ming starts to walk forward 9 steps from the tree location- point A, and sets up point B, and he continues to walk one more step, and he sets up as point C. Big Quoi puts a wooden stick on point B (wooden stick needs to be taller than a person). Connect A and D, Little Ming stands at point C, he looks toward the tree top – point D, his visual line meets on the wooden stick - point E; if he looks straight to AD, his straight visual line meets the wooden stick at- point F. If Big Quoi measures the length *EF* = 25 cm, and *BF* = 160 cm, what is the estimated high of the tree? [Conversion: meters x 3.28 = feet, feet x 0.3048 = meters, centimeters x 0.3937 = inches, inches x 2.54 = centimeters]





Solve: perhaps we can draw a triangle ODG; we then can set up Little Ming's eyes position as point O, and connect a straight line from O to line *AD* at point G.

Because: ∵ *EF* // *DG* ∴ Δ*OEF* ~ Δ *ODG* and (in proportion) *DG*: *EF* = *OG*: *OF* = 10: 1, thus *DG*: 25 = 10: 1, therefore we can estimate *DG* = 250 cm and the high of the tree is *AD* = *DG* + *GA* = 250 + 160 = 410 (cm)

Analysis for Mathematizing Instructional Cycle

This problem can be characterized as having five PISA aspects:

1. Starting with a problem situated in reality.

Locating where the tree is in a real-life setting stimulates students' curiosity to want to know how tall the tree is (applied and authentic).

2. Organizing it according to mathematical concepts and identify the relevant mathematics;

Organizing by setting up three points, the tree top, where Little Ming's vision line meets the location of the tree and the tree top; connect these three points to represent a triangle. Further, an illustration of two similar triangles Δ*OEF* ~ Δ *ODG* is presented.

3. Gradually trimming away reality through processes such as making assumptions about which features of the problem are important, generalizing and formalizing (which promote the mathematical features of the situation and transform the real problem into a mathematical problem that faithfully represents the situation) (OECD/PISA, 2003, p. 27).

The real problem is transformed into a mathematical problem by using the feature of similar triangles.

4. Solving the mathematical problem;

Using the fact that two similar triangles are in proportion, it is easier to measure the height in the smaller triangle ΔOEF and apply the proportion calculation to estimate the height of the larger triangle.

5. Making sense of the mathematical solution in terms of the real situation.

Relating, reflecting, and recognizing—recognize that the distance in nine steps from the tree can form a smaller similar triangle is reasonable to measure the height of the tree. There are no other obstacles between the two triangles or any other factors that may affect the usefulness of the mathematical solution.

Analysis for the Cognitive Demand

It is clear that example 1 fits the definition of mathematical problem-solving in an authentic context. Although the problem was given the strategy to walk 9 steps from the tree and put in a stick for a possible solution, students have to work effectively with an explicit model for complex concrete situations that may involve constraint or call for making assumptions to create triangles that are in proportion. The complexity lies partly in the need to thoughtfully combine information presented both graphically and in text. Moreover, there is no answer that students can see immediately; students need to interpret the graph, and realize that they can resemble the real situation by drawing two triangles to reflect on their solution as it emerges. For instance, students select and integrate different representations, linking them directly to aspects of real-world situations by representing two similar triangles in proportion. Students at this level can utilize well-developed skills and reason flexibly, with some insight into these contexts. They can construct and communicate explanations and arguments based on their interpretations, arguments, and actions.

This activity of making an original problem approach into multiple complex methods shows that this problem fits into the *reflection* competency cluster.

Item type: Open constructed response

Competency cluster: *Reflection*

Difficulty in literacy level*:* Level V (Chapter II, Table I, pp. 11, 12)

Instructional Cycle 2

Book #2 ,Example 1, p. 133: utilizes Cluster sample data to estimate the homogeneous quantity
Mum Sing's house has a big pond; they have raised some fish. After so many years, these fish reproduced many times. Mum Sing was curious to find out how many fish they had in the pond, so, she used the fishnet taking out fish randomly in the pond, she took out total 210 fish; she made a mark in each fish and put the fish back into the pond. After few days, she used the same method to take out the fish. This time, she took out 125 fish; within those 125 fish, 35 fish had the mark she made. Question, what is the estimated number of fish in the pond? 
Objective for Example 1**:** A good collected sample data has similar characteristics of the homogeneous group; therefore, a good collected sample data can represent the characteristics of the homogeneous groups correctly. For example, if we want to estimate how many fishes are in a pond, it depends on the number of fish, in many cases it is not realistic to take out all of the fish in the pond to count them, thus, a good designed Cluster sample data can give us the estimated number of how many fish are in the pond.

Solve: let X represent the estimated number of fish in the pond; within those fish, 210 fish were marked. Then, put in proportion, the marked fish and the total number of fish is 210/X. Few days later, within the 125 fish taken out from the pond, 35 fish were marked, so, the proportion between the marked fish and taken-out fish is 35/125. If the Cluster sample data can represent the homogeneous group, then, two proportions should be similar, it means,
210/x = 35/125,   35 = 125 x 210, X= 750
Thus the estimated number of fish in the pond is approximately 750 fish.

Analysis for Mathematizing Instructional Cycle

PISA Mathematizing can be characterized as having five aspects:

1) Start with a problem situated in reality.

Locating where the pond is in a real-life setting stimulates students' curiosity to want to know how many fish are in the pond (applied and authentic).

2) Organize it according to mathematical concepts and identify the relevant mathematics.

Organizing by understanding the concept in the Cluster sample data to see the relationship between the Cluster sample data and the homogeneous group.

3) Gradually trim away the reality through processes such as making decisions about how to collect sample data for the fish, generalizing and formalizing a similarly complex problem into mathematical form, and transform the real problem into a mathematical problem that faithfully represents the situation. (p. 27)

Thus, the real task is transformed into a mathematical problem by using the feature of the Cluster sample data.

4) Solve the mathematical problem.

Using the fact that twice taking the fish out of pond and the marked fishes are in proportion, it is easier to measure and estimate the total number of fish by solving it in proportion.

5) Make sense of the mathematical solution in terms of a real situation.

Relating, reflecting, and recognizing: recognize that the problem can be simplified by sampling. Relating the sample with a homogeneous group is the key feature in solving this problem. No other factors may be affecting the usefulness of the mathematical solution.

Analysis for Cognitive Demand

The unusual way this authentic problem is presented to students immediately moves it beyond the *reproduction* competency cluster. This involves interpretation and reasoning skills right from the start of the problem. Most students will probably simulate the situation mentally. They will have to find the "clue"—a homogeneous quantity—and be able to identify if the quantity has similar characteristics to the homogeneous group. This activity of making connections between different interpretations does make the problem fit into the *connections* competency cluster. However, the underlying concept of understanding the homogeneous quantity is important not only within the discipline of mathematics but also in daily life.

Some may argue that the context could favor students who have a fish pond in their living environment. It should be pointed out that mathematical literacy includes the ability to use mathematics in contexts different from the local one. That does not necessarily mean that students who live with a fish pond in their environment may not be in a somewhat advantaged position (OECD/PISA, p. 58).[2]

Item type: Open constructed response

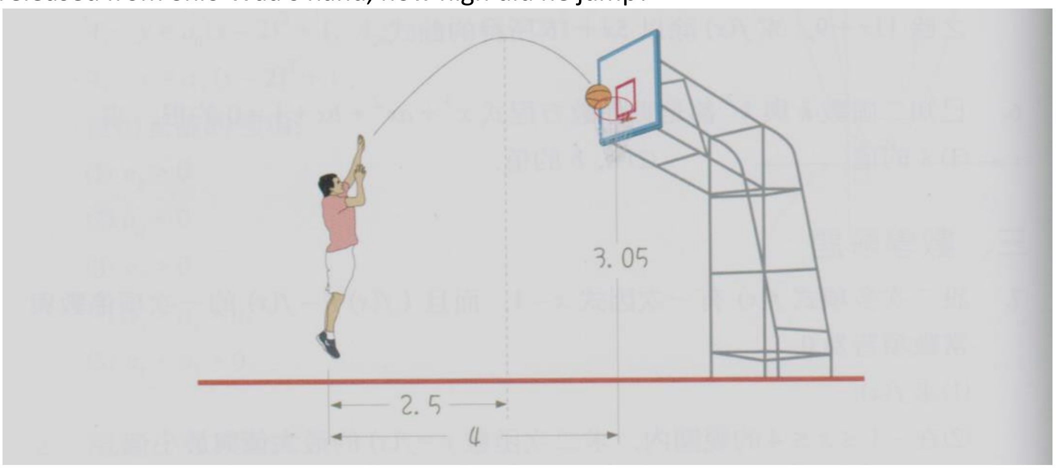
Competency cluster: *connections*

Difficulty in literacy level*:* Level IV

[2]However, the item by country analysis gives no indicator that this is the case here, based on the item example given in OECD/PISA (Mathematics Literacy, p. 58).

Instructional Cycle 3

Book #3, Exercise 10**,** p. 126: This picture shows a basketball court, the distance from the center of basket hood to the ground is 3.05 m, Shio Waa is 1.8 m tall, and he was shooting the ball 4 m from the basket stand under the court. The ball traveled form a high overreaching arch line. When he jumped and shot, at the time he released the ball out of his hand his hand was higher than his head 0.25 m, when the ball traveled through 2.5 m in the air, it reached the maximum point at 3.5 m vertical high, then, the ball continued to travel into the hoop. Question: when the ball was released from Shio Waa's hand, how high did he jump?



Solve: we can use Quadratic function: Standard (Vertex) Form $y = a(x - h)^2 + k$, the maximum point is the vertex.

- If $a < 0$, the quadratic will open downwards.
- If $a > 0$, the quadratic will open upwards.

If the starting point is the time when the ball left Shio Waa's hand, and symmetry for the quadratic is X = 1.5 m (because 4m-2.5m = 1.5 m) , then , Shio Waa was position in left of the point (0, 3.05) by 1 m (-1 m), the other symmetry point is (3,3.05), and the vertex is (0, 3.5) .so Y = K = 3.5

3.05 = *a* (0 – 1.5) ^2 + K …… ①

3.05 = *a (3- 1.5) ^ 2 + K …… ②*

If ① - ②, we get 3.05 = 2.25 *a + K, substitute K = 3.5, we have a= - 0.2, let Y be the high where Shio Waa's position when he through the basketball to the hoop, we already have given his high 1.8 m and 0.25 m at the time he release the ball.*

Y = - 0.2 (-1 -1.5) ^2 + K

Y = - 0.2 (2.5) ^ 2 + 3.5

Y = - 1.25 + 3.5 = 2.25, 2.25 – 1.8 – 0.25 = 2.25 – 2.05 = 0.2 m

Analysis for Mathematizing Instructional Cycle

Mathematizing can be characterized as having five aspects:

1) Begin with a problem situated in reality.

Clearly this example is situated in reality by finding the relation from the given and plot into the mathematics formula **Quadratic: Standard (Vertex) form** $y = a(x - h)^2 + k$ (applied and authentic). This example also requires an understanding the concept of space and distance.

2) Organize mathematical concepts and identify the relevant mathematics.

First, students begin by realizing if $a < 0$, the quadratic will open downwards.

3) Transform the real-world problem into a mathematical problem that faithfully represents the situation.

Next, to transform the real problem into a mathematical problem, students realize that it might be better to set up the total height when Shio Waa jumped and shot for Y.

4) Solve the mathematical problem.

Then, construct the two given relationships between *X* and *Y* to set up two standard (Vertex) form $y = a(x - h)^2 + k$ (solving the problem). Then, students examine the given maximum height and solve for *K*. Next, combine two quadratic functions and solve the unknown *a* in algebraic terms.

5) Make sense of the mathematical solution in terms of the real situation, including identification of the limitations of the solution.

Finally, students work out relative to where *Y* should be situated. The example was given as a description of the real situation; it sets up the sense of how to apply variable *Y* in a real situation and using mathematics to solve the problem, empirically testing the results.

The second stage of Mathematization involves a deductive problem. As soon as students translate the problem into a mathematical form, the process can continue with mathematics. Students pose questions like “Is there ...?”, “If so, how many?”, “How do I find ...?” using known mathematical skills and concepts (three **Quadratic: Standard (Vertex) form** $y = a(x - h)^2 + k$).can be constructed by given). They attempt to work on their model of the problem situation to adjust it (use the first two quadratics to solve the variable *a*, and *Y* for the third *quadratic function* is not the direct answer), to establish regularities, to identify connections, and to create a good mathematical argument (the *Y* is the total height which includes Shio Waa's height, his over-reached hand distance when he threw the ball from his head and how high he jumped). This part of the mathematization process is generally called the deductive part of the modeling cycle (Blum, 1996; OECD/PISA, 2003, p. 39; Schupp, 1988). However, more than strictly deductive processes may play a part in this stage; this part of the mathematization process includes:

- Using and switching between different representations;
- Using symbolic, formal, and technical language and operations;
- Refining and adjusting mathematical models; combining and intergrading models;
- Argumentation;
- Generalization (OECD/PISA, 2003, p. 39).

The last stage is where the mathematization process passes from the mathematical solution to the real solution; and where this is related back to the original real-world problem. These steps (or this step) in solving a problem involve reflecting on the whole mathematization process and the results. Here students must interpret the results with a critical attitude and validate the whole process. Such reflection takes place at all stages of the process, but it is especially important at the concluding stage.

Analysis for Cognitive Demand

The classification of the situation depends, of course, on whether or not people are actually interested in the situation based on their own curiosity to know the answer. One can safely argue that this item is somewhat scientific (because of the use of the formulas), but many sportsmen and women, coaches and trainers do measure how high and how far their jump needs to be in order to reach their goal. Because we are dealing more with modeling, a classification with the *connections* competency cluster seems rather straightforward, as well as the overarching idea change and relationships.

Once we can identify the example to qualify the *connections* competency cluster, we can further analyze whether the example satisfies the *reflection* competency cluster.

This example clearly indicates the complexity of the problem which requires students to think and reason about three given positions under the basketball court: 3.05m, 4m, and 2.5 m. Each location was also given a relatively different height.

The *reflection* competency cluster needs to satisfy the argumentation that includes distinguishing between providing proofs and broader forms of argument and reasoning; recognizing the positions and giving the argument to set up the equations properly before solving mathematically are reflective in this example. In addition, this example requires communication to others on why and how the equations are set up. The communication includes explaining the complex relationships logically. This example also includes reflecting through analyzing, offering a critique, and engaging in more complex communication about models and modeling. Finally, the difference distinguishing between the connections and reflection competency clusters is the reflection on strategies and solutions in problem posing and solving. The strategies to solve this example question by setting two quadratic functions and solving by substitution satisfy the *reflection* competency cluster. From the above descriptions, this example is classified as a *reflection* competency cluster.

Item type: Open constructed response

Competency cluster: *reflection*

Difficulty in literacy level*:* Level VI

Instructional Cycle 4

Book #4, Example 12, p. 31: Wine glasses are set up into 4 levels as shown in the picture.The bottom level is set up as a right triangle and each side has 4 glasses; for every level goes up, each side has 1 less glass. Use this sequence and concept of logic to finish four levels. Question: If we copy the same sequences to set up wine glasses, the bottom right triangle level has 10 glasses in each side, the setup has a total of 10 levels, then, how many glass are needed?

Solve: If each side of right triangle has exactly *K* pieces of wine glass, then, the total number of wine glasses is the sum of numbers start from number 1 to *K* (positive and continue) is ,
1+ 2 + ··· + *K* = *K (K+1)*/2.
From the given question, there is 10 levels of glasses, and the number sequence for the bottom level for each side of right triangle is from 1,2,3,···, 10 pieces wine glasses, thus the total number for 10 levels of glasses is :

$\sum_{k=1}^{10} (k+1)/2$, also

$\sum_{k=1}^{10} \frac{(K+1)}{2}$ = ½ ($\sum_{k=1}^{10}(k^2+k)$) = ½ ($\sum_{k=1}^{10} k^2 + \sum_{k=1}^{10} k$)

= ½ (10(10+1) (2·10+1)/6 + 10(10+1)/2)

= ½ (385 +55) = 220

Therefore, the total numbers of wine glasses needed to prepare are 220 pieces wine glasses.

Analysis for Mathematizing Instructional Cycle

PISA Five aspects of mathematizing are as following:

1) Start with a problem situated in reality.

Locating where the glass setting is in real life stimulates students' curiosity to want to know how many glasses are needed in total versus how many levels of glasses (applied and authentic).

2) Organize it according to mathematical concepts and identify the relevant mathematics.

Students organize by setting up the right triangle on the bottom level and each level goes up with one glass less on each side. They recognize the sequence by representing the sequence in sum of number sequence - $1 + 2 + \cdots + K = K(K+1)/2$.

3) Gradually trim away the reality through processes such as making assumptions about which features of the problem are important, generalizing, and formalizing.

The real challenge is transformed into a mathematical problem by using the feature of sum of number sequence, and recognizing that K represents the number of levels of wine glasses.

4) Solve the mathematical problem.

Students use the fact that each level of glasses is 1 less on each side; thus, for the 4 levels, count the numbers from the top level to the bottom level sequence as 1, 3, 6, and 10. They are 1, $1(1+1)/2$, $2(2+1)/2$, $3(3+1)/2$, and $4(4+1)/2$, Thus, the sum represents the total number of glasses in the four levels of right triangles. By applying the same concept, the sum of 10 levels of glasses is to set up the sum of the number sequence from 1 to 10 and solve it mathematically.

5) Make sense of the mathematical solution in terms of a real situation.

Relating, reflecting, and recognizing: recognize that the numbers 1, 3, 6, and 10 relate to $K(K+1)/2$, and reflecting the question to any level of wine glasses setting. There are no other obstacles between the right triangles or any other factors that may be affecting the usefulness of the mathematical solution.

Analysis for Cognitive Demand

The classification is rather straightforward since this requires an understanding of the geometry and algebraic concept. This is the kind of the problem that one would only come across in a social setting; and it involves reflection because of the rather heavy thinking and reasoning concept. This example requires students to think in detail, and the underlying concepts are rather sophisticated as well. The problem involves the ability to post questions (“How do I know…?”, “How do I find …?”, “What can happen?”, “What would happen if …?”), and the ability to understand and handle mathematical concepts (number series) in contexts that are complex.

The mathematization aspect of identifying the relevant mathematical content and information is important in this example. Superficial reading will lead to misinterpretation. The situation is indeed complex: it varies within how many levels of wine glasses are set up. The variation depends on how many glasses are on each side to hold the set up for the next level. The question asks, “How many glasses are needed?” Thus, it does not require one to fill the middle if there is a hole for one to set up the next level of wine glass. By having a mental image of one missing glass in the middle of a glass tower, students need to come up with a more generalizable strategy involving more mathematical reasoning. Students need to explore in a sophisticated way the relationship between each level of wine glasses to form the summary of glasses in the number series - $K(K+1)/2$, which could be an extremely challenging question for 15-year-olds (p. 70).

Item type: Open constructed response

Competency cluster: *reflection*

Difficulty in literacy level: Level VI

Conclusion and Recommendation

In this study, the investigator used the PISA framework to compare and distinguish between Taiwanese mathematics textbooks. The study demonstrated how textbooks are able to support students’ learning and advancement in PISA’s assessment.

The results indicate a more comprehensive picture of how consistent methodologies are treated in Taiwanese textbooks in relation to PISA. The analyses from the summary indicates that Taiwanese mathematics textbooks do satisfy PISA's Mathematization Cycle—instructional cycles and mathematics functional literacy characteristics. The results of 62.5% [3]same or above cognitive difficulty level V did present the higher level of cognitive demands required by PISA. Additionally, questions did satisfy the description of a *reflection* and *connections* cluster, which were the cluster of higher cognitive demand. Therefore, Taiwanese mathematics textbooks present higher cognitive levels compared with PISA's cognitive demand description.

This study provides evidence and supports the conclusion that Taiwanese mathematics textbooks implement PISA's theoretical basics and are compatible with PISA's mathematics functional literacy. Additionally, this study supports the conclusion that Taiwanese mathematics textbooks contributed greatly to students' success in the international assessment.

Are the selected instructional cycles best exemplified by PISA's theoretical basic contribution to satisfactory results? If "mathematics textbook and functional literacy" *do* influence student learning, could PISA's theoretical bases provide guidelines to help mathematics educators understand more about variations in teaching mathematics? Could the "mathematics textbooks and functional literacy" study extend to other education features?

When interpreting the results of this study, one needs to remember that any education system is complex and involves the interplay of philosophical, historical, and sociological factors. Any analysis of functional literacy needs to recognize the role of all factors such as cultural expectations, family support, government policy and national educational trends. A study analyzing textbooks will, by its nature, tell only part of a larger story.

In Taiwan, access to high school and university is controlled by a series of national exams. Discipline in public schools of all levels is generally very high, with school uniforms and

[3] In the full version of this research paper, have total 13 demonstrations. The 62.5% represents the total percentage from all 13 demonstrations.

morning reveille being the norm. Students of all levels through high school are responsible for cleaning their own classrooms and areas around the school (clean-up time is a daily ritual). Corporal punishment is officially banned, although many reports suggest some teachers still practice it. The language of instruction is Mandarin. Discipline in educational institutions from high school and up (including vocational schools) is the responsibility of military officers stationed at the individual schools (as opposed to elementary and junior high school where teachers and school administrators are responsible for discipline). In addition to the regular academic subjects, students are also required to attend a military education class covering issues such as civil defense, military drills, national defense, and basic firearms training. In the past, high school (and vocational) students were expected to take on civil defense duties in the event of national emergency.

When students enter adulthood in moving from junior high school to senior high school, doing mathematics requires much discipline. This disciplinary aspect of the Taiwanese educational system may very well have an indirect effect on the success of mathematics education. In addition, Confucian thought has set the tone for the learning attitude for over 2,500 years. The philosophy of Confucius emphasizes a strong work ethic in which young students' approach to work and intellectual curiosity are the standards. It is not surprising that during the transitional period from junior to high school, discipline is required in learning mathematics—concentration and focus tend to be much easier for Asian students.

For a future possible "mathematics textbooks and functional literacy" study, the following recommendations are offered:

Improvement of the study: The explanation for all selected instructional cycles' compatibility with PISA's instructional cycle could be that those were the best examples from the four Taiwanese mathematics textbooks. The explanation for why all three instructional cycles only satisfied the *applied* and *authentic* characteristics, could be they are focused on a specific lesson topic. However, the textbooks present higher cognitive levels in comparison to PISA's cognitive demand description. Again, it could be that those are the best examples from the four textbooks. Therefore, this study may not have reviewed instructional cycles that were not as good. A future study should select examples randomly instead of purposely to different results. Moreover, the investigation of instructional cycles and cognitive load could be performed by a judgment panel—perhaps 2-4 mathematics educators, trained by the researcher to provide more objective judgments.

Usefulness of the study for educators: How can the outcomes of this study inform and improve other educators? Will this study be read widely by other educators? *A related, broader question pertains to the extent to which differences in textbooks matter for student learning.* In this study, because the researcher did not study curriculum enactment, it is difficult to answer this question. Yet, given the prevalence of textbooks in schools, differences in textbooks may very well influence what happens in the classroom. Multiple analyses reveal that differences in textbooks potentially affect both teaching and student learning. Thus, future studies could explore if, and how, these differences play out to the extent that teachers can capitalize on how textbooks support mathematics functional literacy. For instance, other educators can develop their own mathematics curriculum using the same PISA methods from this study. Those studies can integrate functional mathematics concepts into lesson texts—exercise and word problems--and develop functional mathematics projects and functioning instructional cycle curricula.

Who makes decisions about the sequencing of topics in textbooks, and how are they made? To what extent are textbooks influenced by available research findings? How do textbooks convey the importance of sequence to teachers? Exploring the textbook authoring process itself is another genre of research. These questions concern which textbook authors across different counties capitalize on available research findings. Can a comparative study be conducted between, for example, Taiwan's and Finland's mathematics textbooks? Can a comparative study be investigated between mathematics textbooks from The People's Republic of China and Taiwan (Republic of China)?

PISA's theoretical basic frameworks provide a set of tools that can be modified by researchers and adapted to address particular needs regarding textbook analyses, yet offer a common language that facilitates comparisons across those analyses. The findings here resonate with this perspective in which textbooks can structure different learning opportunities for students. The findings in the direction of "mathematics textbooks and functional literacy" are suggestive of the richness of the field of cross-national textbook analysis and its potential for understanding and enhancing what contributes to teaching and, ultimately, to student learning.

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