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Engaging Students Through Real-Life Mathematics: Police Station Problem

Pinchback, Carolyn
Long Le
Dennis Show
University of Central Arkansas
Department of Mathematics

Carolyn Pinchback
Le Long
Dennis Show
Department of Mathematics
University of Central Arkansas

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Synopsis

When a city wants to build a new police station, the question that needs to be asked is where the best location to minimize the response time to a call when one comes in. This is the question that we will answer using a technique called method of random search. By analyzing the district breakdown of the city, the frequency of calls to each district, and the gridding of the city, we can answer the question, and begin to see how teaching secondary mathematics can be more meaningful, more engaging, and ultimately, more real.

Engaging Students Through Real-Life Mathematics: Police Station Problem

Dennis Show, Long Le, Carolyn Pinchback

University of Central Arkansas

Abstract

When a city wants to build a new police station, the question that needs to be asked is where the best location to minimize the response time to a call when one comes in. This is the question that we will answer using a technique called method of random search. By analyzing the district breakdown of the city, the frequency of calls to each district, and the gridding of the city, we can answer that burning question, and begin to see how teaching secondary mathematics can be more meaningful, more engaging, and ultimately, more real.

Introduction

The role of the educator has changed dramatically. We are no longer just lecturers. We are artists, entertainers, mentors, confidants, etc., and as to be expected, as our role has changed so has the methods by which we instruct. With the change in mathematics on a national level with many states not only adopting the Common Core State Standards (CCSS), but also already phasing them into the curriculum, for the learner to be successful, there will be a need for the mathematics to be understood on a deeper level. With these CCSS, patterned after NCTM's Principles and Standards for School Mathematics, we want to enhance the learners' understanding of the content; to impel them to not only find a correct answer, but to know *why* their answer is correct or incorrect. The CCSS have eight standards for mathematical practice:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Where might these standards be explored? One answer is problem-based learning. "For students to *learn to use* mathematical knowledge in meaningful ways, they need planned instructional opportunities that connect the concepts and techniques studied to other areas of mathematics"

(Johanning, 2010). Often times the learner is trying to apprehend the merit or importance of the material, but can't comprehend the whole picture. Problem-based learning can give the learner the picture that is needed to help understand the question, "Why do I need to know this?" This problem-based learning, or “real-life” mathematics, can engage the learner and allow him/her to see a connection not only with mathematics to the real world, but within itself as a subject, a vertical connection.

In this paper, we will consider a problem called “Police Station Problem,” (Meerschaert, 2007) and discuss how such a problem can be used with CCSS. Every city has an emergency service, such as police, that when called, has to be dispatched from some location, and get there quickly. “What is the best location to minimize the response time when a call comes in?” is a natural question one may ask. Answering this question is a very important task, especially when a rapidly growing city is looking to build a new police station and needs to determine its new location. This is the problem that we chose to analyze for the city.

Method

This problem is a multivariate optimization problem, with the objective function being the response time. The procedure we chose to solve this problem is by a method of random search. The method of random search takes randomly generated points in the feasible region and compares the objective function evaluated at those points to obtain the one point with the minimum response time.

The Data

Furthermore, the frequency of calls assigned to each district had to be reassigned to each of the new smaller districts. Once this was completed, the city district breakdown went from 10 districts to 84, and provided a much better distribution of the frequency of calls within the city.
Table 1 shows the breakdown of calls to each of the smaller districts.

(Table 1 about here)

Once the data were collected, a grid was needed to overlay the map of the city. The size of each cell in the grid was determined by the map. Some of the streets downtown provided the perfect square size to base the whole grid. From this grid, a coordinate system was introduced with the origin in the lower left corner of the map and the unit length equal the side of a cell. We represented locations on the city map by coordinates (x, y), so the feasible region for this problem was $0 \leq x \leq 23$, $0 \leq y \leq 18$. Picture 1 shows how the city is broken down into smaller districts on an aerial view, and how the city is gridded.

(Picture 1 is about here)

To use the call data, we assumed that the calls were uniformly distributed throughout each district. With this assumption, the number of calls coming from each cell in the grid was determined by dividing the number of calls of each district by the number of cells it covered. To analyze the response time, and not having to consider all of the cells, we selected a set of reference points on the map and allocated calls to those points. We wanted to pick enough points that the call data we had would be viable but not so many that the analysis would be too time consuming. So we considered blocks of 2 x 2 cells, starting from the origin, and chose the points to be the centers of the blocks. Each point now represented the whole block with the total number of calls from all four cells assigned to it. Table 2 is an excerpt from how the calls were distributed to each coordinate.

(Table 2 about here)

Analysis

The optimal location of the police station was the point with the smallest average response time to the other places in the city. In this problem, only the time to the reference points obtained in the previous step was considered because those reference points represent the area surrounded by them. The response time to one call from a location (x, y) to a reference point $(x_i, y_i)$was given as $3.2+1.7r_i^{0.91} = 3.2+1.7\sqrt{(x-x_i)^2+(y-y_i)^2}^{0.91}$ (Meerschaert, 2007), where r was the distance between the location and the reference point. The 3.2 represented the average amount of time to dispatch an officer, and the $1.7r^{0.91}$represented the relationship between the distance to the call location and the time it took to drive there. So if (x, y) was the location of the new police station, the average response time to a call was where f_i was the frequency of calls assigned to the reference point(x_i, y_i). The method of random search considered many randomly generated locations for the police station and selected the one with minimum average response time.

There were many ways to generate random numbers, including rolling dice and using a calculator. Anticipating others may want to use this lesson and its accessibility, we chose Excel even though other spreadsheet programs are equally easy to use. Though, with Excel, *x* can be randomly generated from 0 to 23, and *y* can be randomly generated from 0 to 18. This creates a randomly generated point in the feasible region that represents a possible location for the police station. Excel cannot handle the objective function directly due to a summation of 53 square-roots, the function must be broken up into smaller terms. The distance to every reference point ($f_i\sqrt{(x-x_i)^2+(y-y_i)^2}^{0.9}$) was calculated individually, then all those numbers were summed together in the objective function to yield the average response time. We generated 4000 random points and determine the one with smallest average response time. That was our answer to the Police Station problem. Since it was easy to generate patches of 4000 random locations in Excel, we were able to repeat this process to check the consistency of our answer. Even though the answer changed each time, the locations were created by change. After all we found that there is an optimal region for the location of the police station as shown in picture 2.

(Picture 2 about here)

So, in conclusion, the police department is currently located exactly where it needs to be because there is nowhere else in the radius that it could be built.

Discussion

There are a few points that need some discussion. The objective function that was used came from a textbook and was for a specific example, so there is no guarantee that it works for every city. The Police Station problem can be extended to other emergency response services, such as fire department, ambulance sub stations, or even pizza delivery, and these problems require different values for the parameters. To determine the right parameters for each scenario, more research and data analysis are needed. These can be done as statistics projects.

From a standpoint of CCSS and addressing the eight standards of mathematical practice, we believe that we exhausted the list.

1. **Make sense of problems and persevere in solving them.** We first had to figure out exactly what the problem was (best location for the police station) and then persevere in the analysis. EXCEL couldn't handle a function so large so it had to be broken down and worked on in pieces.
2. **Reason abstractly and quantitatively.** We have several thousand points on a map that represent possible locations for the police station, and surrounding each chosen point are calls that work into determining the average response time, the lowest of which, becomes the best location.
3. **Construct viable arguments and critique the reasoning of others.** There were some decisions that were made that we felt were in the best interest of solving the problem in a timely manner, i.e., time span for the data, even distribution of calls in each district, etc.
4. **Model with mathematics.** We gridded off the map, found an objective function, and gathered data, all of which went into this method of random search to solve the problem.
5. **Use appropriate tools strategically.** Mathematica, Maple or other software could have been used, but we chose a tool, Excel, because of its accessibility.
6. **Attend to precision.** The location is fairly precise. We had to locate the best possible location, and determine if its current location is within an acceptable range.
7. **Look for and make use of structure.** The objective function gives and averages response time; and while taking a derivative and setting equal to zero seems like a simple solution, it actually becomes impractical in solving this problem.
8. **Look for and express regularity in repeated reasoning.** One answer may not have been good enough, but through multiple trials, we found that there was regularity in the best location for the police station.

Education is not static: there is always what appears to be a better way to teach, think, and/or reach the learner. "Making curriculum choices, creating tasks for students, and gauging the appropriateness of problems and activities for students are central components of effective teaching" (Cuoco, Benson, Kerins, Sword, and Waterman, 2010, P 181). To choose the right problem, the right context to teach mathematics can be a challenge for teachers. The Police Station problem is a contextualized problem that can inspire students to learn the "why" in mathematics. According to Rhem (1998), this type of problem-based learning guides students toward meaning-making over fact collecting, hence achieving higher levels of comprehension. While this problem may not fit in any particular class, its solution requires knowledge in many subjects. Random search method using Excel may be beyond the scope of high school, but any method to generate random numbers (such as rolling dice) will work. From Algebra I and Algebra II, we have evaluating exponential functions. There are 53 distance equations for every 1 random point, which can be taken and worked by any of 8th grader in Algebra II. From Geometry, we have gridding a map, and setting a coordinate system for the city. A Statistics class could collect data that is needed in the construction of an objective function for this problem, as they would need frequency of calls, drive distance to a call location, and time taken to drive to a call location. So a teacher can describe the problem in general, and only discuss about the parts that is involved in a particular subject, or the problem can also be used as a big project that requires effort from multiple classes.

Mathematics is not just formulas, equations, and procedures. That is a part of what mathematics is, but the larger picture is to foster and enhance a student's critical-thinking and problem-solving skills, because while a student may never again need to know the quadratic formula or how to find the area of a trapezoid after high school, he/she will always need to know how to think and reason and work with a group to discuss the best way to solve a problem. *That* is the best lesson we can teach them and *that* is the best idea they can take from us educators into the real world.

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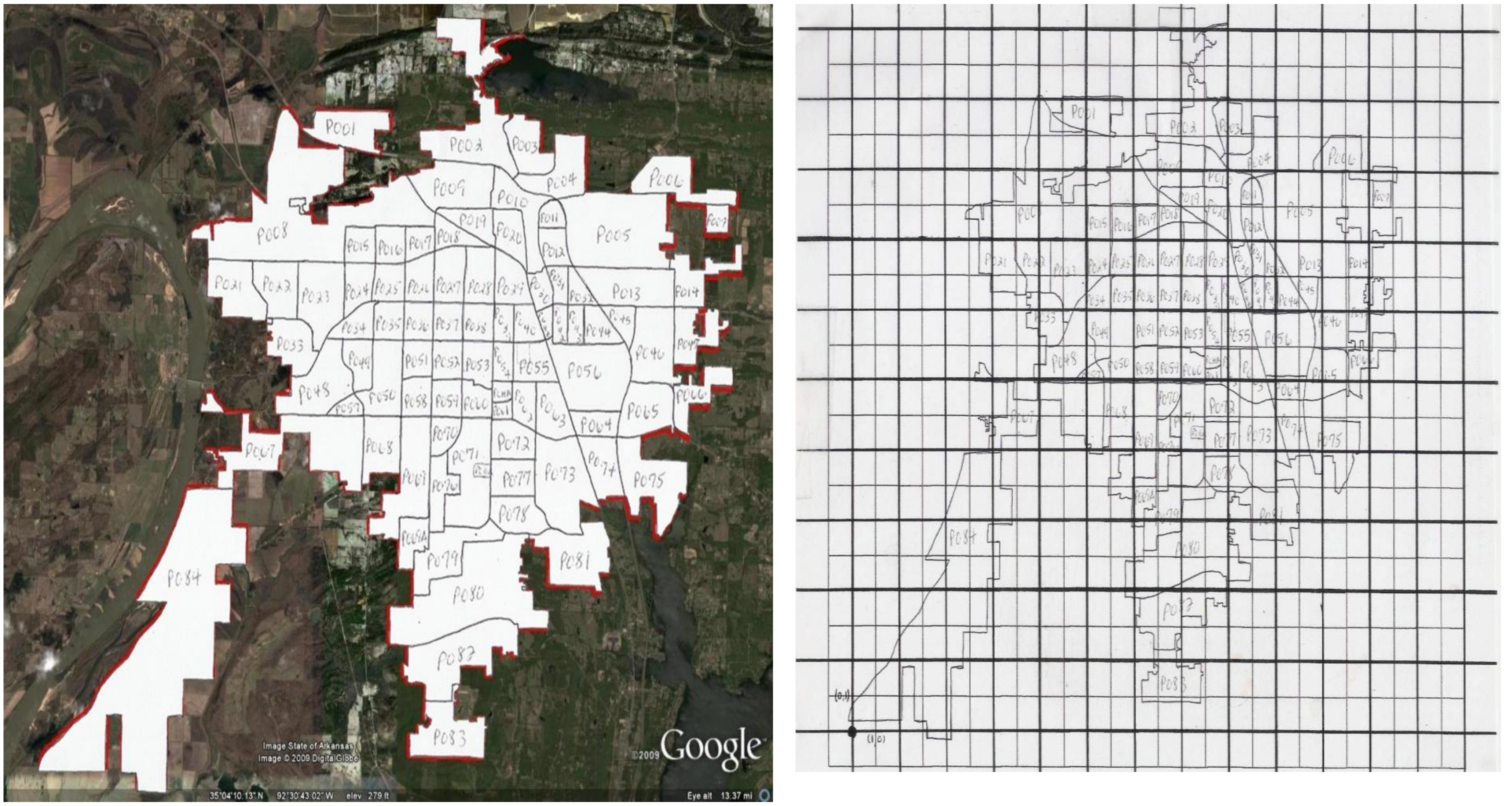
Table 1.

P001	**P002**	**P003**	**P004**	**P005**	**P006**	**P007**
5	98	21	137	227	33	2
P008	**P009**	**P010**	**P011**	**P012**	**P013**	**P014**
379	215	475	291	102	197	119
P015	**P016**	**P017**	**P018**	**P019**	**P020**	**P021**
110	37	54	94	126	216	4
P022	**P023**	**P024**	**P025**	**P026**	**P027**	**P028**
71	174	110	76	110	261	245
P029	**P030**	**P031**	**P032**	**P033**	**P034**	**P035**
361	233	219	369	79	123	17
P036	**P037**	**P038**	**P039**	**P040**	**P041**	**P042**
105	115	205	105	263	392	152
P043	**P044**	**P045**	**P046**	**P047**	**P048**	**P049**
277	305	302	423	122	50	55
P050	**P051**	**P052**	**P053**	**P054**	**P055**	**P056**
126	161	182	149	153	343	419
P057	**P058**	**P059**	**P060**	**P061**	**P062**	**P063**
157	56	157	33	179	298	219
P064	**P065**	**P066**	**P067**	**P068**	**P069**	**P070**
148	112	31	85	31	133	159
P071	**P072**	**P073**	**P074**	**P075**	**P076**	**P077**
341	172	67	98	49	58	118
P078	**P079**	**P080**	**P081**	**P082**	**P083**	**P084**
96	3	20	4	38	20	2
Total Calls: 12,678						

Table 2.

District	84	84	83	83
Coordinate	(1, 1)	(3, 1)	(13, 1)	(15, 1)
No. of Calls	0.272727	0.363636	13	7
District	84	84	82	82
Coordinate	(3, 3)	(5, 3)	(13, 3)	(15, 3)
No. of Calls	0.272727	0.363636	22	16
District	84	80	80	
Coordinate	(5, 5)	(13, 5)	(15, 5)	
No. of Calls	0.363636	7	10	
District	84	69, 79	78	78, 81
Coordinate	(5, 7)	(13, 7)	(15, 7)	(17, 7)
No. of Calls	0.363636	97	167	70



Picture 1

Picture 2