



2017 HAWAII UNIVERSITY INTERNATIONAL CONFERENCES
SCIENCE, TECHNOLOGY & ENGINEERING, ARTS, MATHEMATICS & EDUCATION JUNE 8 - 10, 2017
HAWAII PRINCE HOTEL WAIKIKI, HONOLULU, HAWAII

THE ART OF MATHEMATICS:
EXPLORING TESSELLATIONS WITH
TECHNOLOGY & TRADITIONAL METHODS

CROWE, CHERYLL
DEPARTMENT OF MATHEMATICS
ASBURY UNIVERSITY KENTUCKY

The Art of Mathematics: Exploring Tessellations with Technology & Traditional Methods

Cheryll E. Crowe, Ph.D.

Asbury University

Abstract

Tessellations can be used to engage K-16 students in thinking deeply about the art of mathematics, specifically the intersection of beauty and transformations. In this session, participants will create tessellations using paper/pencil, pattern blocks, GeoGebra (free, open source software), and online applets. Regular, semi-regular, and Escher-style tessellations will be explored through each of these methods. Participants will leave the session with materials to effectively implement the activities in the K-16 classroom.

Keywords: tessellations, transformations, GeoGebra

The Art of Mathematics: Exploring Tessellations with Technology & Traditional Methods

A tessellation, often called tiling, is defined by filling a plane with repetitions of figures without overlap or gaps. Three types of tessellations are often evident in the K-12 classrooms: regular, semi-regular, and Escher-style tessellations. Tessellations are not explicitly outlined in the Common Core State Standards for high school mathematics, but explorations on the topic of tessellations can develop reasoning ability, critical thinking, and creativity skills in students (Nirode, 2013). While the art of mathematics may be seen as an optional curricular component, the importance of mathematical aesthetics in fostering children's mathematical thinking can be foundational, as students' aesthetic criteria often motivates their mathematical explorations (Eberle, 2014).

In grades K-12, students often explore tessellations using the traditional methods of paper/pencil and pattern blocks (Chick et.al, 2008; Holly, 1995). A set of pattern blocks consist of green triangles, orange squares, blue parallelograms, tan rhombi, red trapezoids, and yellow hexagons. The tactile approach to using pencil/paper and pattern blocks can helpful for students of all ages to explore tessellations; however, there are restrictions to the use of these tools, specifically given the limited number of shapes available with pattern blocks. Therefore, the use of technology to also explore tessellations can provide a rich learning experience to students that they would not be able to access using tactile manipulatives. This paper will address ways to teach regular, semi-regular, and Escher-Style tessellations using traditional methods and technology.

Exploring Regular Tessellations

Regular tessellations are produced from tiling regular polygons. Since the regular polygons must fill the plane at each vertex, then the interior angles must be an exact divisor of

360°. To explore regular tessellations, only one shape of pattern blocks is used to determine if a tessellation can be created. Through hands-on discovery learning, students are able to determine that only three of these polygons produce a tessellation in the Euclidean plane, and all three are regular polygons. Teachers often have students trace the pattern blocks using pencil and paper to create a tessellation that fills an entire page.

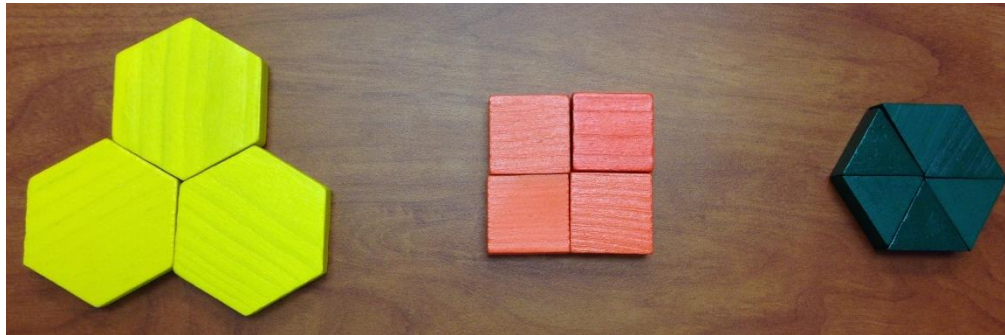


Figure 1. Regular Tessellations Using Pattern Blocks

The use of geometry software can also foster interesting explorations and aid in visual connections. Through the use of the “Regular Polygon” tool, students can create a variety of polygons and then rotate the polygons to determine if a tessellation is created. The angle measurement tool is particularly useful in making connections to the types of regular polygons that tessellate.

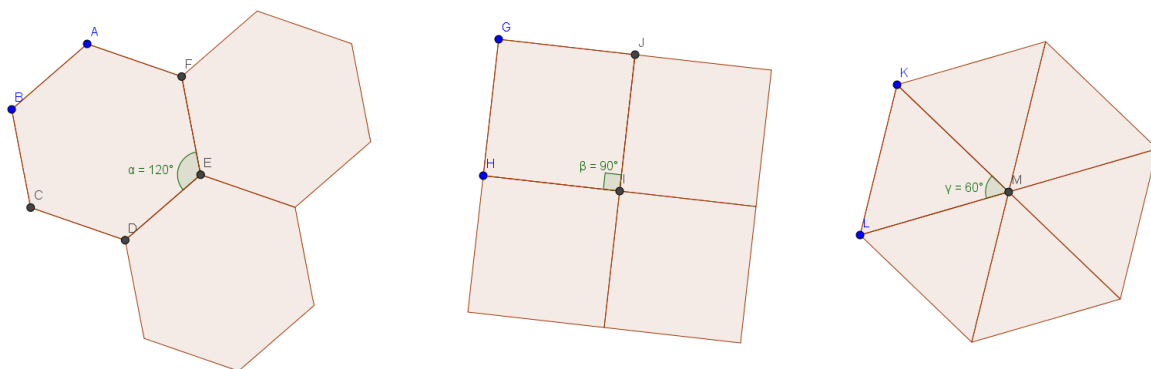


Figure 2. Regular Tessellations Produced in GeoGebra

These pre-images or initial figures can be translated within the software to produce larger tessellations that fill the GeoGebra screen. Also, adding color and pattern to the polygons could add interest to the sketch and foster mathematical discussion.

Exploring Semi-Regular Tessellations

Semi-regular tessellations include more than one type of regular polygon, and the arrangement of polygons at each vertex is the same. The naming/numbering system for semi-regular tessellations is based on counting the number of sides of the polygons around the given point of rotation. Of particular interest at the K-12 level is the exploration of the arrangements of regular polygons that produce semi-regular tessellations. Here again, pattern blocks can provide an opportunity for discovery and enable students to consider various groupings around a given vertex. Two common semi-regular tessellations are shown in Figure 3.

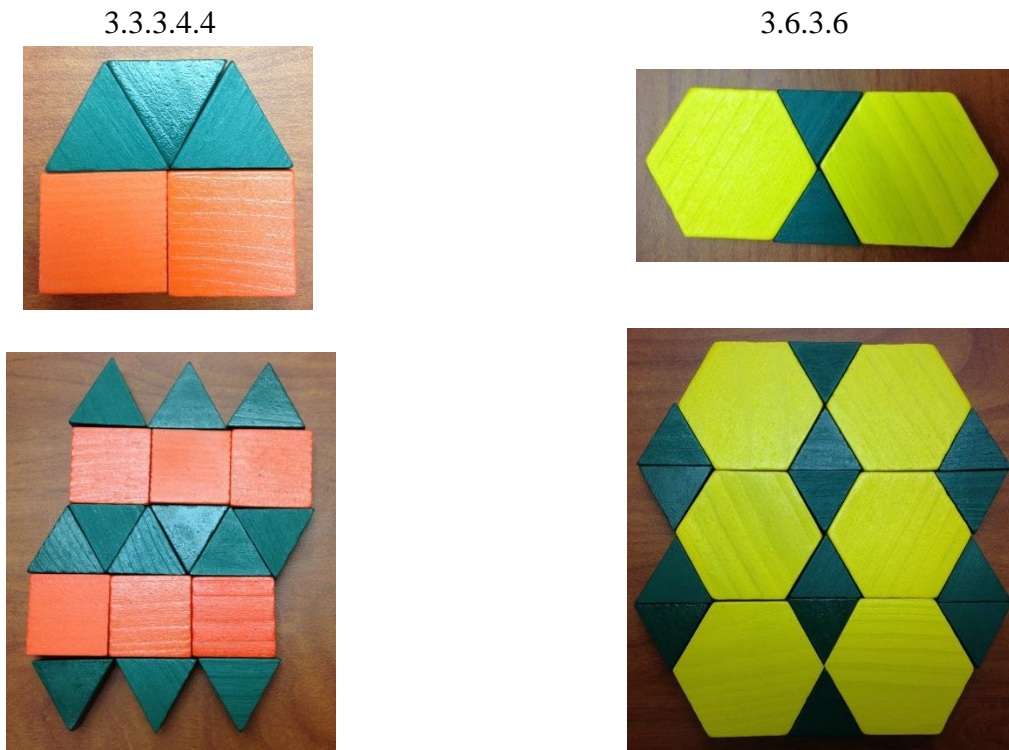


Figure 3. Semi-Regular Tessellations Using Pattern Blocks

While pattern blocks are helpful, the exploration can be cumbersome for more complex semi-regular tessellations. Also, semi-regular tessellations that utilize octagons or dodecagons cannot be created given the limitations of the shapes of pattern blocks. Therefore, it is challenging for students to discover all eight semi-regular tessellations using manipulatives. However, by incorporating the use of geometry software, students can have a rich learning experience and explore all eight of these tessellations. Again, the use of the “Regular Polygon” tool in GeoGebra is invaluable in the exploration of semi-regular tessellations. As illustrated in Figure 4, all eight semi-regular tessellations can be created and manipulated during the software.

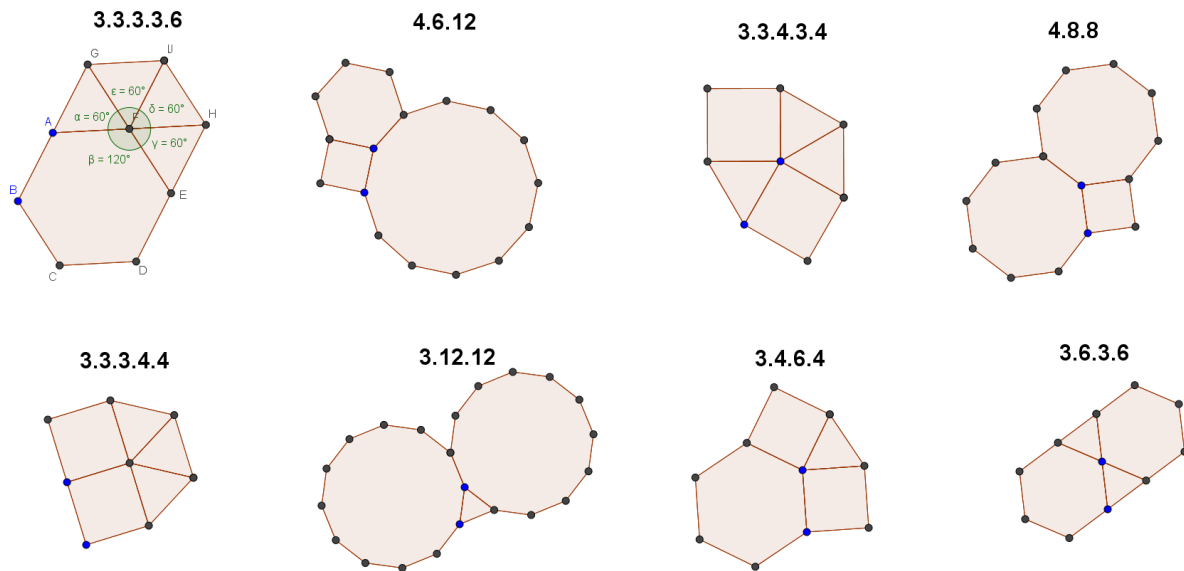


Figure 4. Pre-Image for Semi-Regular Tessellations Produced in GeoGebra

These pre-images can be translated using the tools in GeoGebra to produce large tessellations that fill the entire screen. As evident in 3.3.3.6, the use of the “Angle Measurement” tool can provide a visual connection to the sum of the angles of 360° at a given vertex. Similar to the exploration with regular tessellations, color and patterns within the polygons could also provide clarity as the large tessellation is created.

In addition to GeoGebra, online applets can provide a meaning mathematics exploration of tessellations for students. Links to several web-based generators for regular and semi-regular tessellations are available in Appendix A. Each of these applets focus on the properties of tessellation and encourage student discovery of tessellations.

Exploring Escher-Style Tessellations

Escher-style tessellations are often the most revered tessellations in the K-12 classroom as these tessellations truly demonstrate the “art of mathematics” and engage students in interesting investigations. With a background in graphic art, M.C. Escher became fascinated by the regular Division of the Plane and greatly impacted the field of mathematics through his work in tessellations (MC Escher Company, 2016). Simplistic Escher-style tessellations can be produced by hand using a fairly efficient process; directions for classroom implementation are available in Appendix B.

In order to explore more complex designs, online applets or GeoGebra can be very effective for student exploration related to Escher-Style Tessellations. Directions to create an Escher-Style Tessellations using GeoGebra is available in the Appendix C. While very engaging, the process in GeoGebra requires precision and can become time consuming, especially for students with special needs. An alternative to GeoGebra is to design tessellations using online applets which eliminates some of the cumbersome steps so students can participate in class exploration and discussion more efficiently. A list of online applets to foster the creation of Escher-style tessellations is available in Appendix A.

Conclusion

Whether discovering tessellations using traditional methods or with technology, students in grades K-12 who engage in such activities can gain valuable insights into many mathematical concepts. The explorations outlined in this paper certainly address the Standards for

Mathematical Practice as outlined in the Common Core State Standards. Through these activities with tessellations, students address six of the eight practice standards (CCSS, 2016):

- Make sense of problems and persevere in solving them.
- Reason abstractly and quantitatively.
- Model with mathematics.
- Use appropriate tools strategically.
- Attend to precision.
- Look for and make use of structure.

However, perhaps the most significant objective is not found within formal curriculum: fostering an appreciation for the “art of mathematics”.

References

- Chick, L., Holmes, A. S., McClymonds, N., Musick, S., Reynolds, P., & Shultz, G. (2008).
Geometry and measurement. *Teaching Children Mathematics*, 14(7), 409-409.
- Common Core State Standards Initiative [CCSS] (2016). *Standards for Mathematical Practice*.
Retrieved from <http://www.corestandards.org/Math/Practice/>
- Eberle, R. S. (2014). The role of children's mathematical aesthetics: The case of tessellations.
Journal of Mathematical Behavior, 35, 129-143.
- Edkins, J. (2007). *Design a Tessellations Online (Escher-Style)*. Retrieved from
<http://gwydir.demon.co.uk/jo/tess/sqtile.htm>
- Holly, K. (1995). Shape up! *Teaching Children Mathematics*, 2(4), 226.
- Math is Fun (2013). *Tessellation Artist*. Retrieved from
<http://www.mathsisfun.com/geometry/tessellation-artist.html>
- M.C. Escher Company (2016). *About Escher*. Retrieved from: <http://www.mcescher.com/about>
- National Council of Teachers of Mathematics (2016). *Tessellation Creator*. Retrieved from
<https://illuminations.nctm.org/Activity.aspx?id=3533>
- Nirode, W. (2013). Don't sacrifice geometry on the common core altar. *Mathematics Teacher*,
107(3), 168-170.
- NRICH (2016). *Tessellation Interactivity*. Retrieved from <https://nrich.maths.org/6069>
- Petti, W. (2016). *Tessellation Town*. Retrieved from
<http://www.mathcats.com/explore/tessellationtown.html>
- Shodor (2016). *Tessellate!* Retrieved from
<http://www.shodor.org/interactivate/activities/Tessellate/>

Appendix A: Online Tessellations Applets

Regular & Semi-Regular Tessellations

Tessellation Creator: <https://illuminations.nctm.org/Activity.aspx?id=3533> (NCTM, 2016)

Tessellate!: <http://www.shodor.org/interactivate/activities/Tessellate/> (Shodor, 2016)

Tessellation Artist: <http://www.mathsisfun.com/geometry/tessellation-artist.html>

(Math is Fun, 2013)

Tessellation Interactivity: <https://nrich.maths.org/6069> (NRICH, 2016)

Escher-Style Tessellations

Design a Tessellations Online (Escher-Style): <http://gwydir.demon.co.uk/jo/tess/sqtile.htm>

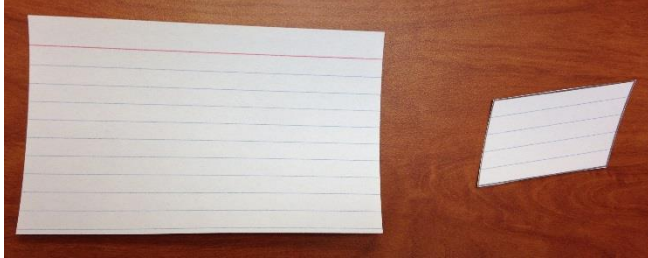
(Edkins, 2007)

Tessellation Town: <http://www.mathcats.com/explore/tessellationtown.html> (Petti, 2016)

Appendix B: Directions to Create an Escher-Style Tessellation using Traditional Methods

Step #1:

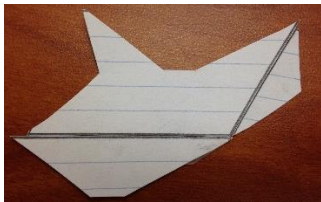
Begin with a standard 3"x 5" index card. Cut out a quadrilateral (preferably parallelogram, rectangle, rhombus, or square) as the original figure.

**Step #2:**

Cut a design from the left side of the figure and then perform a horizontal translation of the piece to the right side of the figure. Tape the piece to the quadrilateral.

**Step #3:**

Cut a design from the top side of the figure and then perform a horizontal translation of the piece to the bottom side of the figure. Tape the piece to the quadrilateral.

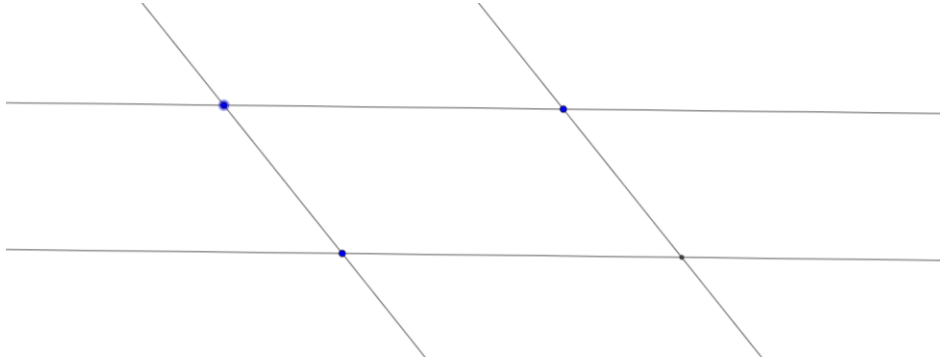
**Step #4:**

You have now created the pre-image for your tessellation. Using transformations, fill an 8 ½" x 11" sheet of paper without any white space or overlap between figures.

Appendix C: Directions to Create an Escher-Style Tessellation using GeoGebra

Step #1:

Construct a parallelogram.



Step #2:

Using the polygon tool, create the parallelogram and then hide the lines.



Step #3:

Using the segment tool, construct a random, simple shape on the left side of the figure.



Step #4:

Construct a vector that is the length and direction of the top or bottom side.



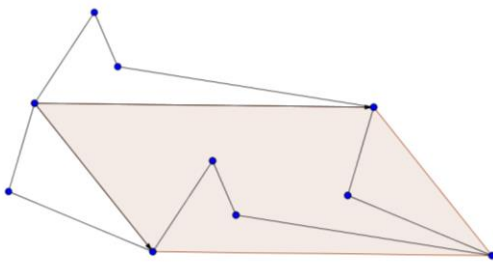
Step #5:

Use the translate vector tool to translated each segment of the random shape.



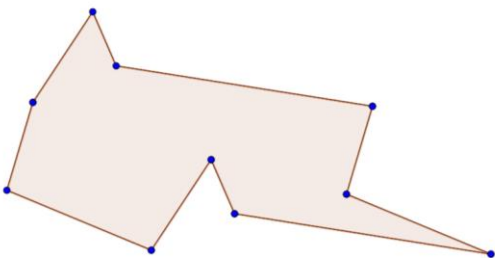
Step #6:

Repeat the process outlined in steps #3-5 to construct a second random, simple figure at the top and bottom of the figure.



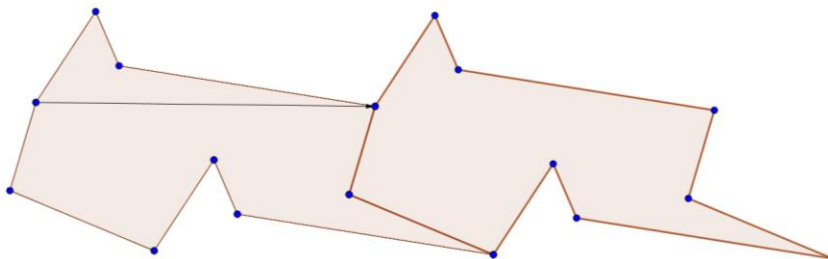
Step #7:

Hide the two vectors and the original parallelogram. Then connect the segments and points using the polygon tool. This object will be the pre-image for the tessellation.



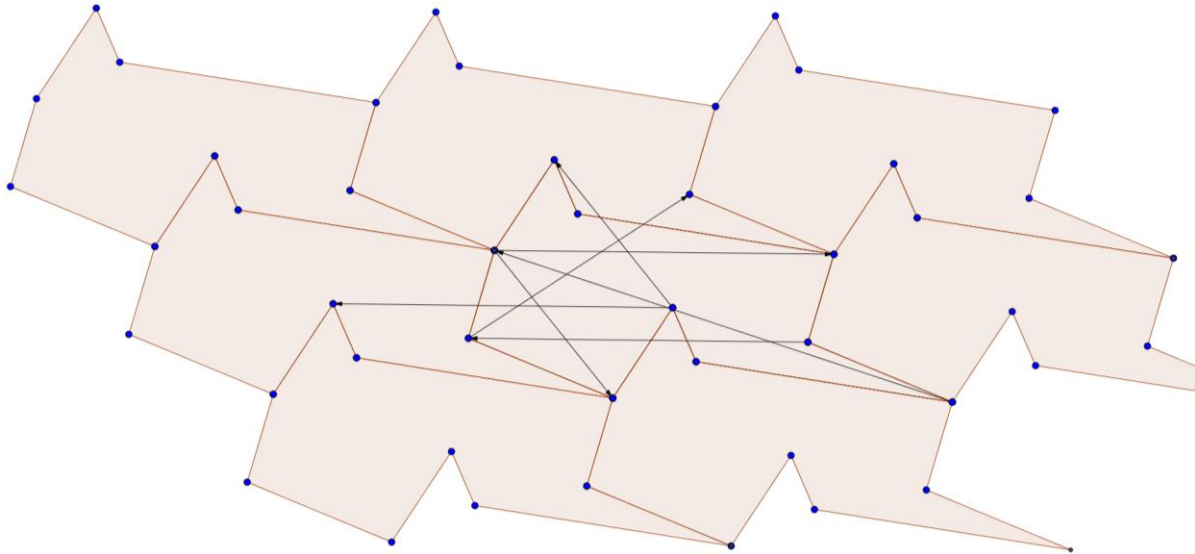
Step #8:

Use the vector tool to create a vector from one point to another point in the figure in order to translate the figure. Then translate by vector, selecting the pre-image and then the vector.



Step #9:

Repeat the process with various vectors to translate the figure in the plane to create an Escher-Style Tessellation (Note: You will need to consider the direction of the vectors in order to correctly perform the translations necessary to achieve the characteristics of a tessellation.) You can change the color as well as the pattern of the figures.

**Step #10:**

Hide the vectors and then adjust the color and pattern of each polygon as desired.