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STEAMED MUSIC

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Synopsis:

This paper will present how STEAM can be useful to better understand and to explore music. It will cover topics from mathematical modeling of sound, musical tone systems, statistical analysis of musical compositions, to exploring new styles of creating music using computing. These topics are chosen from an interdisciplinary general education science course on STEAM and music, which the author designed and taught for USC Upstate's Honors program.

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1. Introduction

This paper tries to increase understanding and appreciation of the relationship between STEAM and music. It will show how STEAM can be useful to analyze, manipulate, and create music. STEAM is important to music. It provides answers and solutions to the many questions related to the science of sound, technology of sound recording, engineering of instruments and other musical equipment, artistic creation of new styles of music, mathematical modelling of sound, and statistical analysis of musical compositions. Computers, the Internet, and technology have advanced the way we learn. Playing sounds, displaying graphics, viewing moving images, and programming solutions have enhanced conceptual understanding. With the use of technical computing software, we can tackle real data and real problems. This paper especially tries to highlight the endless possibilities of algorithmic music programming in creating new styles of music that challenge traditional music and that do not even require playing an instrument.

2. Using STEAM to Analyze, Manipulate, and Create Music

This chapter demonstrates how we can use STEAM to analyze, manipulate, and create music.

Music is an artistic composition of sounds. In order to understand music, we first need to understand sound. What makes a pleasing sound, like the sound of a violin? What makes an unpleasing sound, which we call noise? Technical computing enables us to play a sound whose amplitudes are given by digits, for example, the digits of the decimal number representations of real numbers. When we convert the digits of the decimal number representations into amplitudes of sounds, we discover that rational numbers like 1/19 will create pleasing sounds and that irrational numbers like π will create unpleasing sounds. We recognize that the decimal number presentation of 1/19 has a repeating group of 18 decimals (526315789473684210). Hence 1/19 could be written as $0.0\overline{526315789473684210}$, where the over-lined region indicates that this particular group of digits repeats itself over and over. We discover that this cyclical behavior is crucial for the creation of a pleasing sound. It is similar to the vibrating behavior of a plucked guitar string or a violin string played by a bow. Irrational numbers such as π do not have any repeating group of decimals and hence sound like unpleasant noise. When we experiment with the number of digits used in a repeating group and change individual numbers to study the effect on the sound, we recognize that manipulating the length of the repeating group of decimals (1/19 has 18 repeating numbers) influences the pitch or frequency of the sound. The more numbers that are listed in the repeating group, the lower the pitch is; and the fewer numbers that are listed, the higher the pitch is. Sound is usually measured in Hertz (Hz), the number of repeating cycles in one second. If we play A4 on a tuned piano, the string vibrates 440 (Hz) times back and forth in one second. If we keep the length of the repeating group of digits the same but change the individual numbers in the group, the timbre or character of the sound changes but the pitch stays the same.

If asked what the digital recording of a plucked guitar string or a bowed violin string looks like or what tones were played, we realize that the study of trigonometric functions is useful. With technical computing, it is possible to inspect the data in a digital audio file. When we analyze the relationship between the presented numbers, graph them, and try to find patterns, we rediscover the cyclical behavior; and we can determine the main frequency, *freq*, (or pitch) of the recorded tone that was played by the violin. The simple sine function Sin(*freq**2*π*t) represents a simple mathematical model of the recorded violin tone. Sin(*freq**2*π*t) has the same frequency as the recorded violin tone. When we convert that sine function into sound and compare it to the recorded sound, we find that the pitches are identical but the characters (or timbres) differ. To create a more accurate mathematical model where even the timbres are identical, we need to learn about Fourier series to calculate sums of cosine and sine functions that perfectly match the data of the digital audio recording, (Fig. 1).

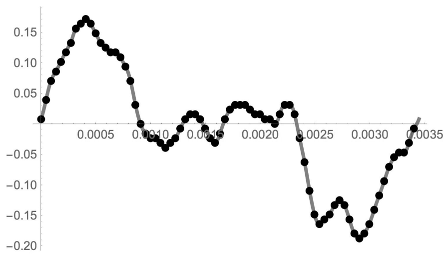

Fig. 1. The Fourier series matches perfectly the data of the digital audio file.

The Fourier series involves sine and cosine functions that are integer multiples of the main frequency, *freq*. These integer multiples are called harmonics and determine the character (or timbre) of the violin. Using technical computing, we are able to compute Fourier series, a task that would have been extremely cumbersome if done by hand.

Contemporary music of the Western world is based on the twelve-tone equal tempered musical tone system. All the instruments in a symphony orchestra are designed according to this system so that they can play in tune and harmony with each other. In the study of the twelve-tone equal tempered musical tone system, we investigate the relationship between the frequencies of the individual keys of the piano (Fig. 2).

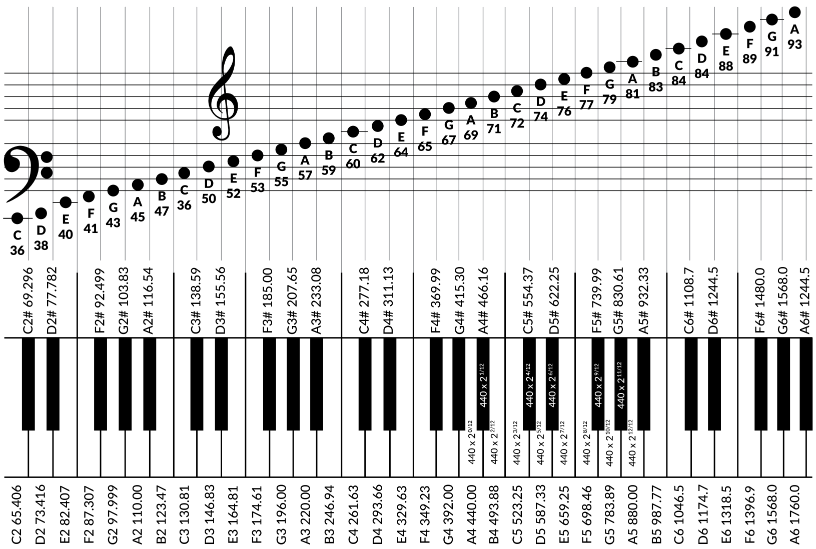

Fig. 2. Twelve-tone equal tempered musical tone system and MIDI numbers.

After examining these relationships, we realize that the number 2 is as fundamental to sound as π is to anything circular. To be more specific, the octave of any tone has twice the frequency of the original tone. For example, the frequency of A5 (the octave of A4) is 880 Hz, which is 2 times the frequency of A4 (440 Hz), (Fig. 2). The change in frequency between two successive tones in the 12-tone equal tempered system is determined by the factor $2^{1/12}$. For example, the frequency of A4 is 440 Hz, and the frequency of the next higher tone A4# is 466.16 Hz, which is $440{*}2^{1/12}$. The frequency of B4 is 493.88 Hz, which is $440{*}2^{1/12}{*}2^{1/12}$. Now we can apply the knowledge we have learned about frequencies by learning how to place frets on a guitar and how to create 24- or 36-tone equal tempered musical tone systems, which are used in other cultures around the world.

Can we challenge the traditional musical systems? What are the limitations of the 12-tone equal tempered musical tone system? Certain musical compositions outside the twelve-tone system are impossible to play by a traditional symphony orchestra. By using technical computing, however, we can go beyond the traditional twelve-tone equal tempered musical system. We can create pure tones of any frequency and then use them to design complex compositions that originate from geometrical shapes, for example. Specifically, we can convert the coordinates of the famous butterfly curve (Fig. 3) into audible frequencies of music that is no longer a culmination of discreet individual tones but rather an instantaneous change in frequencies. Also, we can translate certain points on a three-dimensional shape into a discrete musical composition (Fig. 4).

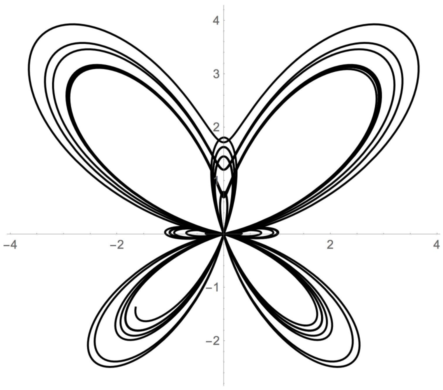

Fig. 3. Butterfly Curve.

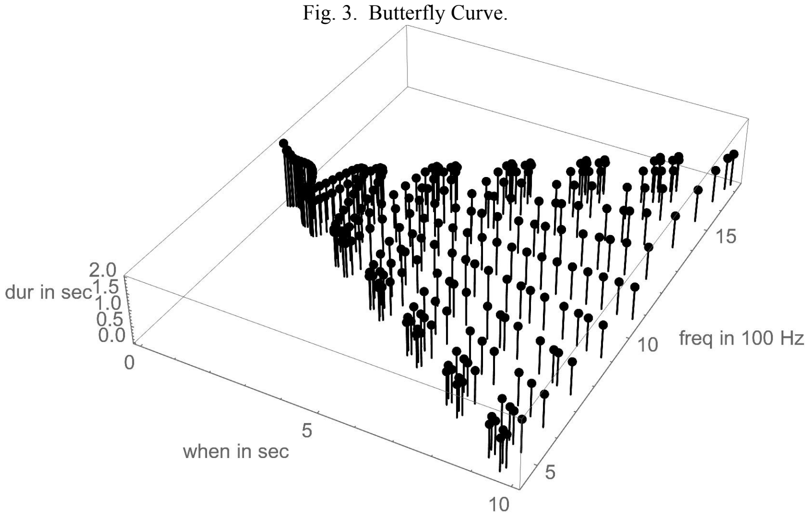

Fig. 4. Discrete musical composition that originates from a geometrical shape.

Music is available online in the form of MIDI files. For classical music MIDI files, I use the website *Kunst der Fuge* ("Kunst der Fuge," n.d.). MIDI stands for Musical Instrument Digital Interface. A MIDI file is a collection of all the individual tones played in the composition. Each tone contains the following information: the pitch, the time when the tone is turned on, the time when the tone is turned off, the instrument, and the volume. Each tone in the 12-tone equal tempered musical tone system is associated with a MIDI number

(Fig. 2). For example, C4 is associated with the MIDI number 60, C5 with 72, A4 with 69, A5 with 81, and so on. Each available instrument is associated with a MIDI number. For example, the Acoustic Grand Piano has MIDI number 1, and the Fretless Bass has MIDI number 36. Please consult with the MIDI Association ("MIDI Association," n.d.) for all the details. In a MIDI file, the information to play C4 on the Acoustic Grand Piano after one second for 0.25 seconds as loud as possible is coded as {60, {1, 1.25}, 1, 1}, and to play A5 on the Fretless Bass after 1.5 seconds for 1 second half as loud would be {81, {1.5, 2.5}, 36, 0.5}. By means of technical computing software, it is possible to access the data of any MIDI file and hence access the information of each individual tone in the musical piece. Once we have access to this information, it is possible to perform a statistical analysis of the data. We can now compare the music of different composers. For example, Table 1 compares Beethoven's piano sonatas with Mozart's piano sonatas.

Table 1. Comparison of Piano Sonatas

Average	Beethoven	Mozart
Song length in seconds	1001	825
Tones played	9047	6326
Mode	61	68
Different tones	64	55
Range	65	58
Mean	63	67
Standard deviation	13	11

Once we have access to all the data in a MIDI file, it is not too complicated to manipulate the data and to create variations of the original musical piece. For example, a piano sonata could be played backwards and reflected where individual tones are reflected with respect to the mean. Mathematical integer functions can be mapped to individual notes to completely change the character of a musical piece. We can even produce our own integer sequences and use them to create our own MIDI files. The creative possibilities are unlimited.

References

1. Kunst der Fuge. (n.d.). Retrieved from http://www.kunstderfuge.com/.
2. MIDI Association. (n.d.). Retrieved from https://www.midi.org/.