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THE VOLIVOLI FISH: MATHEMATICS EMBEDDED IN COMMERCIAL DESIGN

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The Volivoli Fish: Mathematics Embedded in Commercial Design

Synopsis:

This paper describes the development of a mathematically-rich task with multiple entry levels, inspired by the geometry and algebra embedded in a commercial logo. The author's experience in researching the use of technology in the secondary mathematics classroom, along with many years of teaching mathematics to students of a variety of age and ability levels, led him to discover a number of possible geometric and algebraic investigations for students, enhanced through the use of CAS technology.

The Volivoli Fish: Mathematics embedded in commercial design

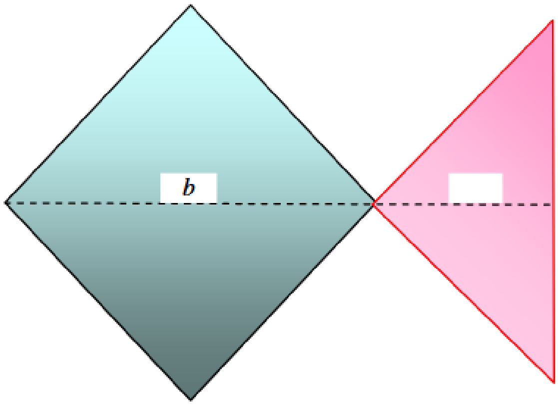
Introduction

Those of us who are teachers of school mathematics in Australia, compared with colleagues in many other countries, enjoy a high level of freedom regarding our ability to choose classroom activities for our students. Unencumbered by state- or nationwide overly prescriptive curriculum, we can address the perceived learning needs of our students using a range of resources and stimuli. This paper describes the development of a mathematically-rich task with multiple entry levels, inspired by the geometry and algebra embedded in a commercial logo.

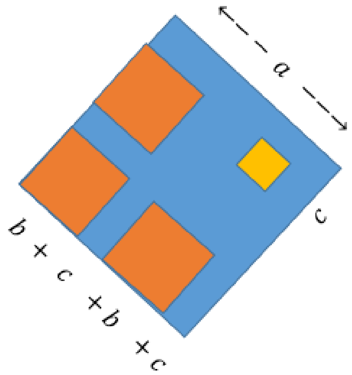
The links between mathematics and art, where symbolic and visual representations of abstract ideas provide sources of rich intellectual exploration, are well documented in the literature and lately, promoted through the emphasis on STE**AM** in school curriculum. Betts & McNaughton (1995) explore the interconnection between mathematical and artistic endeavours, citing the Golden Ratio and Escher's drawings as being amongst several better-known examples. Their paper also cautions against an over-emphasis on algorithmic procedures in school mathematics curricula, inviting teachers to embed aesthetic appreciation of observed phenomena into classroom activities. Griffiths (2005) provides a framework for identifying the components of a rich task in mathematics education as centred on explorations of a logo used by the Fibonacci Association. His call for students to "actively …participate in the construction of their own knowledge" (p217) is one best answered when teachers can enable a variety of technologies and hands-on learning activities in their classrooms.

Silver (1997) states that the nature of mathematical thinking and the discipline of mathematics have, at their core, problem posing and solving. Thus, teachers whose lessons can reflect their own journey in discovering, formulating and solving problems of genuine interest are modelling the essence of being mathematicians for their students. The challenge as perceived by the author is for teachers to channel their own curiosity regarding problem exploration into meaningful and ability-appropriate tasks which allow students to pose the same or similar "What if..?" questions as were first experienced by their teachers.

The author's previous experience, in being part of the research team which developed the lesson materials for Marina's Fish Shop (Wander & Pierce, 2009), provided a springboard for curiosity when the logo at the centre of this investigation was first viewed in Fiji in 2015. In Marina's Fish Shop, the algebra emerging from a dynamic geometry representation of a fictitious fish-shaped sign led to analysis of the embedded quadratic function. The commercially developed Volivoli Fish logo, used for directional signage as well as brand recognition, was thought to have its own set of mathematical properties which invited further investigation. Figure 1 shows representations of each "fish" and describes the underlying mathematical ideas for each.



(a) Marina's Fish Shop sign: Fish body and tail are formed by a square and right-angled triangle, respectively. Total horizontal width =10m, fish body width = *b* m; the total area is to be maximised as *b* varies. See HREF1.

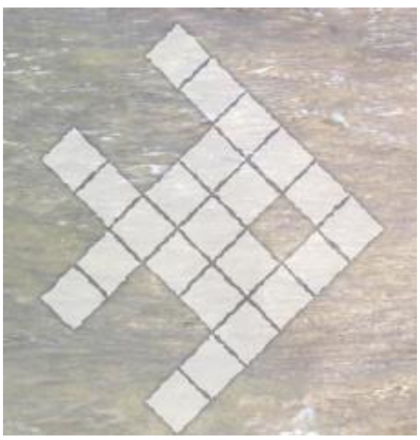


(b) Volivoli Fish logo: Three medium and one small square are removed from large square, forming the fins and the eye of the fish, respectively; $a = 2b + 2c$; the percentage of large square area remaining is to be investigated as the ratio $b : c$ varies.

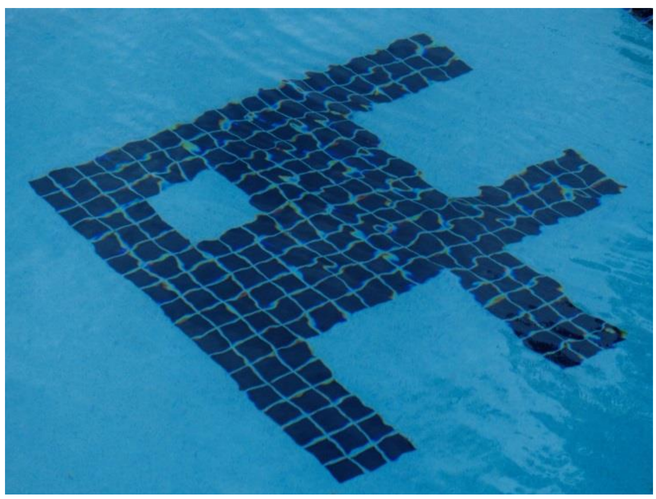
Figure 1: Fish-related geometric diagrams and the nature of the investigations

The Problem

The author was on a holiday in Fiji where it was noticed that a resort was using a logo meant to represent a stereotypical tropical fish. This logo appeared in varying sizes with some appearing on promotional material, directional signage, furniture and walls. Others were seen submerged in the tiling of the swimming pool's base and steps. All representations, whether in or out of the water, showed the square tiling pattern which enabled an area calculation to be completed (ignoring the width of the grouting or drawn border lines) using simple arithmetic of counting tiles. See Figure 2.



Promotional material (see HREF2)



Tiling on base of swimming pool

Figure 2: Representations of the Volivoli Fish logo, each showing b:c = 2:1

Problem 1 was posed and could be easily solved: What percentage of the original "large square" remained after the four smaller squares were removed?

A more careful examination of a photograph of another below-water tiling pattern from the deep end of the pool revealed a different ratio was now being used. See Figure 3.

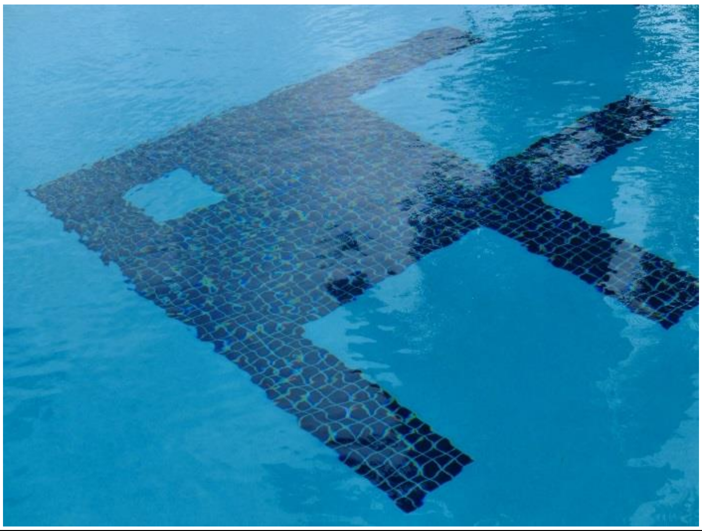


Figure 3: b:c = 3:1

In light of this discovery, it was decided to classify those Volivoli Fish with $b:c=2:1$ as Type I, and those with $b:c=3:1$ as being Type II. Problems 1&2 had identical wording regarding the percentage of original large square area remaining after the removal of the four squares for each of Types I and II.

When calculations unsurprisingly revealed different percentage values, and that Percentage (Type I) > Percentage (Type II) >50%, the author looked at a suddenly more complex Problem 3: What ratio *b*:*c* is required for exactly 50% of the original large square to remain?

Pedagogical questions then surfaced as the author considered the artistic and mathematical benefits arising from students using physical materials to create their own fish, and using technology to model the likely complex equations arising from Problem 3.

Multiple pathways

It was evident that, given a sufficient amount of time and access to the "old" technology of the ruler, graph paper and scissors, students could design their own fish for any suitable (ie, easy to handle) square of side length *a* cm, provided the relationship $a=2b+2c$ was maintained. Alternatively, e-technology such as that provided by Geogebra® could assist a CAD approach. In either way, students' freedom to explore a ratio of their choice and make the necessary area calculations should ensure rich discussion of techniques and discoveries. These activities are ideal for late primary/elementary to middle secondary students who need application work to maintain engagement in standard curriculum topics such as area, measurement, ratio, similarity and arithmetic. However, senior students will also benefit from an initial hands-on approach before attending to the abstractions involved in exploring the algebraic formulations which emerge.

Creative students who look at extreme ratio values may provoke interesting (and valid) aesthetic-based reactions from peers who may dispute the fish-like look of some of these creatures. Of course, the possibilities are entirely in the hands of the experienced teacher who designs the task to fit the needs and abilities of the students.

The role of technology

The author used TI-Nspire CAS technology to construct flexible Type I and Type II Volivoli fish as can be seen in Figure 4 below, where a Geometry application was used. These diagrams featured sliders to vary the value of *a*, and preserved the proportionality of *b*:*c* required for each of Types I and II. Teachers can use these to point out the essential unifying feature of all Volivoli fish diagrams; that is, the fact that $b+c=\frac{1}{2}a$.

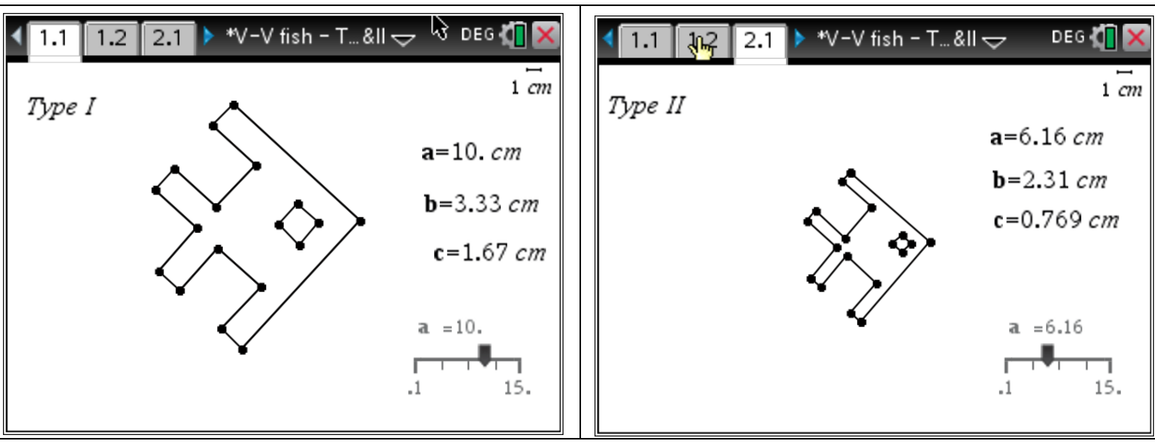
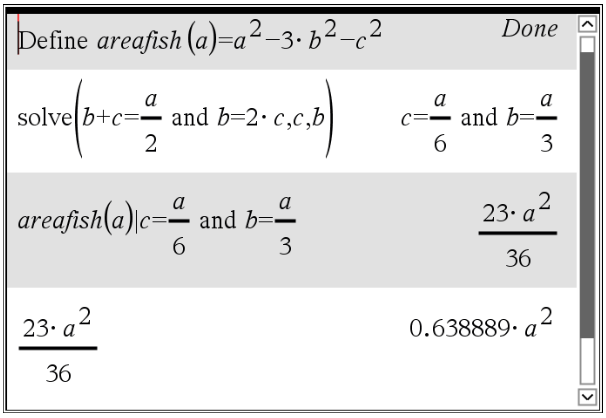


Figure 4: Screenshots of dynamic geometry used in Nspire CAS to illustrate Types I&II properties

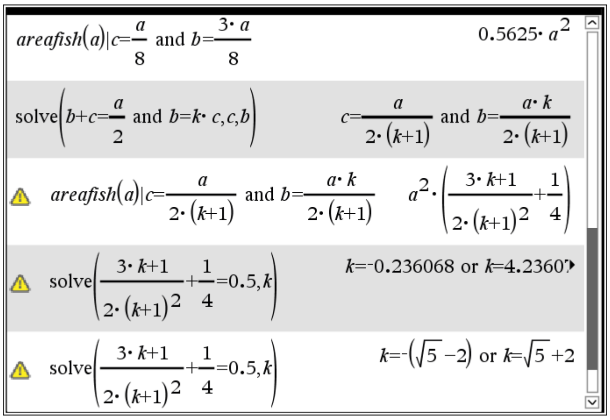
Intended as a pedagogical device for teacher demonstration of similarity, the Nspire files developed for this activity have other pages (Calculator and Graphs applications) which the teacher can use to prepare students for Problem 3. Alternatively and hopefully, students can develop their own pages to complement or replace by-hand algebra for further analysis. This will of course be dependent on their technology exploration confidence and competence levels. See Figures 5 and 6 below for detailed explanations.



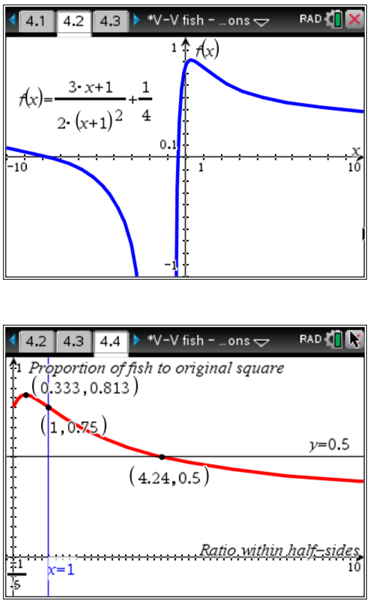
1st: The function showing the remaining area as a function of *a* is defined as *areafish*
2nd: The relationship that variables *b* and *c* have to *a* and the proportional relationship between *b* and *c* for Type I fish allow us to form simultaneous equations, enabling *b* and *c* to each be defined in terms of *a*.
3rd: The function *areafish* is re-expressed in terms of one variable (*a*), showing the exact fraction of the original square remaining in a Type I fish
4th: The fraction is converted to a decimal, implying the solution to Problem 1 is ~64%.

Nspire Calculator application page	Explanation of steps shown on this page

Figure 5: Screenshot and explanation of 4 procedural steps of CAS solution to Problem 1 (Type I)

	1st: For Type II fish, the ratio 3:1 allows a similar calculation, showing the solution to Problem 2 is ~56%. 2nd: The variable *k* is introduced as the unknown value of *b*:*c*, resulting in both *b* and *c* to be re-expressed in terms of *a* and *k*. 3rd: The function *areafish* is re-expressed showing the exact fraction of the original square remaining in any fish where *b*:*c* = *k*. 4th and 5th: That fraction is set = 0.5, and k is found in both approximate and exact forms; thus the solution to Problem 3 is that a ratio of $(\sqrt{5}+2):1$ is required for a Type III fish.
Nspire Calculator application page	Explanation of steps shown on this page
Figure 6: Screenshot and explanation of 5 procedural steps of CAS solutions to Problems 2 and 3	

The solution to Problem 3 (that the ratio *b*:*c*, or *k* needed to be larger than 4:1) then leads to further questions: Is there an upper limit to the value of *k*? What happens when the restriction that *b* > *c* is relaxed so that any positive ratio is considered? At what point does the *k*-value alter the aesthetics of the logo so that its resemblance to a fish is lost? Technology allows us to explore the situation a bit further by replacing *k* with *x* and exploring the function $f(x)=\frac{3x+1}{2(x+1)^2}+\frac{1}{4}$ in its graphical representation; see Figure 7 below.

	The function, showing the proportion of the original large square remaining as a function of the ratio *b*:*c* with an unrestricted domain, is one which senior secondary students analyse through calculus using by-hand algebra and technology. However, its application to the actual Volivoli fish logo requires the ratio to be positive… Graph analysis shows that the maximum percentage of the large square (~81%) remaining will occur when *b*:*c* = 1:3, for which the eye of the fish will appear disproportionately large and strangely located. The point (1, 0.75) indicates that when *b* = *c* there is 75% remaining, and the point (4.24, 0.5) represents the Type III fish. Asymptotic behaviour as *b*:*c* becomes large suggests a lower limit of 25%, though any such fish would have a tiny eye and impossibly thin fins (perhaps falling victim to natural selection?)
Nspire Graphs application pages	Interpretation of graphical representations of the function
Figure 7: Screenshots of graphical analysis of the function representing the ratio b:c	

Thus, analysis of area remaining can occur on a number of levels, where the artistic implications of the logo as physically constructed can be expressed in mathematical terms. Extension work for students (again, upper-primary/elementary to senior secondary) could focus on specifics related to perimeter for this same shape.

Conclusion

For the author, one of the satisfying aspects of conducting this relatively simple mathematical exploration has been experiencing continual "What if…?" questions being posed as initial results were obtained. This would appear to be consistent with the notion that creative activity is often generated by the posing and solving of problems (Silver, 1995). It is hoped that teachers who enable their students to explore the mathematical properties of artistic creations will be similarly inspired to promote an atmosphere of creative flexibility (as described in Griffiths, 2010) within their classroom. Mathematical ideas often start with real world experiences, and it is hoped that the examples discussed in this paper will generate similar rich learning activities for teachers and students alike.

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HREF1. Marina's Fish Shop. New Technologies for Teaching Mathematics. (2009). Retrieved from: https://extranet.education.unimelb.edu.au/DSME/RITEMATHS/general_access/curriculum_resources/quadratic_functions/Marinas%20fish%20shop/MarinasFishShop_Worksheet.pdf

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