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THE FASCINATING MAGICAL APPLICATIONS AND PROPERTIES OF ARITHMETIC SEQUENCE

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The Fascinating Magical Applications and Properties of Arithmetic Sequence

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ABSTRACT: A sequence is a set of numbers in a specific order. The two simplest sequences which are interesting to work with are the classical arithmetic and geometric sequences. Since arithmetic and geometric sequences are so nice and regular, they have simple and friendly formulas. They have many interesting mathematical properties which are enjoyable and have exciting mathematical patterns.

In mathematics, an arithmetic sequence is a sequence of numbers such that the difference of any two successive members of the sequence is a constant called common difference. For instance, the sequence 3, 5, 7, 9, 11, 13... is an arithmetic sequence with common difference 2.

If the initial term of an arithmetic sequence is A_1 and the common difference of successive members is *d*, then the *n*th term $\mathbf{A}_n$ of the sequence is given by:

$$\mathbf{A}_n = \mathbf{A_1} + \mathbf{(n - 1)d}$$

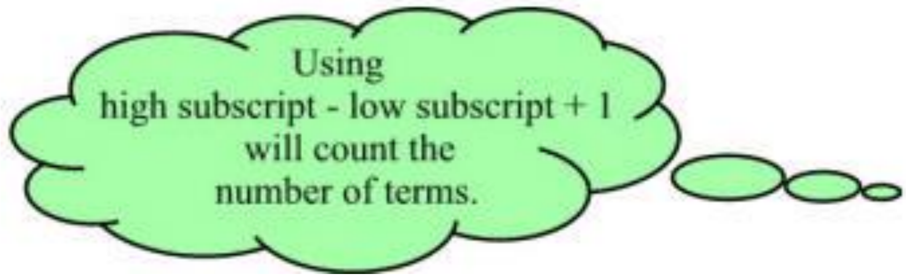
The arithmetic sequence has very interesting properties and keep popping up in many places as illustrated below.

2. Background Materials: The Following facts and main results will be listed here and used as quick references (a). To find any term of an arithmetic sequence we use**:**

$$\mathbf{A_n = A_1 + (n - 1)d}$$

(b) Generally, to check whether a given sequence is arithmetic, one simply checks whether successive entries in the sequence all have the same difference.

(c) Number of terms is: $(A_n - A_1) + 1$ **or**



(d) An **arithmetic mean** is the term between any two terms of an arithmetic sequence. It is simply the average (mean) of the given terms.

((i) **Dirichlet's Theorem on Primes in Arithmetic Sequence** (1837)

If A_1 and d are relatively prime positive integers, then the arithmetic sequence A_1, A_1+d, A_1+2d, A_1+3d, ... contains infinitely many primes.

(J) The arithmetic series is the *sum* of the numbers in an arithmetic sequence.

(k) The triangular numbers are numbers formed by partial sum of the arithmetic sequence 1,2,3,4, 5, ….n . In other words, triangular numbers are those counting numbers that can be written as $T_n = 1+2+3+\ldots+ n = \frac{n(n+1)}{2}$ [2]

3. SOME PROBLEM SOLVING TECHNIQUES USING ARITHEMATIC sequence

Next, we will illustrate some fascinating problem-solving techniques applying the properties and patterns of arithmetic and geometric sequences.

Problem 1. Give **a**n example of two arithmetic sequences that contain infinitely many primes.

Solution: This is possible by Dirichlet's Theorem**.** The following are the easy ones to consider.

(a) 1, 7, 13, 19, 25, 31, 37, ...
(b) 5, 11, 17, 23, 29, 35, 41, ...

Note tha**t** together these two sequences seem to contain all of the primes except 2 and 3

Problem 2. Find A_{15} for an arithmetic sequence where

$A_3 = -7 + 3k$ and $A_6 = -15 + 13k$.

Solution: Note that:

$$A_6 = -7+3k+3d$$

$$\Rightarrow -15 + 13k = -7+3k+3d$$

$$\Rightarrow \mathbf{3}d = -8+10k$$

Now, we have $A_{15} = -7+3k+3d$

$$= -7+3k-8+10k$$

$$= -15+13k.$$

<u>Problem 3.</u> Savannah State Herty Hall Conference Room has square tables which seat four people. When two tables are placed together, six people can be seated. If 5000 square tables are placed together to form one very long table, how many people can be seated?

Solution: The pattern that is emerging is clearly an arithmetic **s**equence. The numbers in the sequence begin **4, 6, 8, 10, ...** .

Using $\mathbf{A}_n = A_1 + (n - 1)d$ with $n = 5000, d = 2$ and $A_1 = 4$,

we get the 5000th term = 4 + [(5000 - 1) X 2]

= 4 + [4999 X 2]

=4 + 9998

= 10002

Therefore, 10002 people could sit at 5000 tables.

<u>Problem 4</u>. A company began doing business four years ago. Its profits for the last 4 years have been $11 million, $15 million, $19, million and $23 million. If the pattern continues, what is the expected profit in 30 years

Solution: The pattern that is emerging is clearly an arithmetic **s**equence. The numbers in the sequence begin 11**, 15, 19, 23, ..** . Using $\mathbf{A}_n = A_1 + (n - 1)d$ with $n = 30, d = 4$ and $A_1 = 11$,

we get the 30th term = 11 + [(30 - 1) X 4]

= 11+116

= 127

Therefore, 127 million will be the profit in 30 years.

Problem 5. Find the number of multiples of 9 between 17 and 901.

Solution: Note that using arithmetic sequence formula we have

$900 = 18 + 9(n - 1)$ and solve for n.

$$864 = 9n - 9$$

$$873 = 9n$$

$$99 = n$$

There are 99 multiples in the given range.

Problem 6. Three numbers form an arithmetic sequence, the common difference is 11. If the first number is decreased by 6, the second is decreased by 1, and the third number is doubled, the resulting numbers form a geometric sequence. Determine the three numbers that form the geometric sequence.

Remark. A geometric sequence is a sequence of numbers where each term after the first is found by multiplying the previous one by a fixed non-zero number called the common ratio. For example, the sequence 2, 6, 18, 54, ... is a geometric sequence with common ratio 3. Similarly, 10, 5, 2.5, 1.25, ... is a geometric sequence with common ratio 1/2

Solution: Three numbers in A.S. can be written:

A, A + d, A + 2d.

If the d is 11, write;

A, A + 11, A + 22.

If the first number is decreased by 6,

write A - 6

the second is decreased by 1,

write A + 10

and the third number is doubled

write 2A + 44

If three numbers form a geometric sequences. then there is a common ratio:

$$\frac{\text{Second}}{\text{First}} = \frac{\text{Third}}{\text{Second}}$$

$$\frac{A + 10}{A - 6} = \frac{2A + 44}{A + 10}$$

$$A^2 + 20A + 100 = 2A^2 + 32A - 264$$

$A^2 + 12A - 364 = 0$

$(A + 26)(A - 14) = 0$

$A = -26$, $A = 14$

If A = - 26, the A.s. is -26,-15,-4 and the G.S. would be:

-32, -16, -8, so your common ratio is 2.

If A = 14, the A.S. is 14, 25, 36 and the G.S. would be:

8, 24, 72, so your common ratio is 3.

4. Main Results

Next, we look at some of the important theorems in the mathematics of the arithmetic and geometric sequences

Theorem 1. The sum s_n of the first n terms of an arithmetic sequence is given by

$$\mathbf{S_n} = \sum_{k=1}^{n} \left(A_1 + (k-1)d\right) = \mathbf{n/2(A_1 + A_n)}$$

Proof: Note that S_n can be expressed in two ways as follows:

(1) $S_n = A_1 + (A_1+d) + (A_1+2d) + (A_1+3d) + \ldots + (A_1+(n-3)d)$

$+ (A_1+(n-2)d) + (A_1+(n-1)d)$

(2) $S_n = (A_n - (n-1)d) + ((A_n - (n-2)d)) + (A_n - (n-3)d) + \ldots + (A_n - 2d) + (A_n - d) + A_n$

Now Adding (1) and (2), we get

$$2S_n = n\left(A_1 + A_n\right)$$

$$\Rightarrow S_n = \frac{n\left(A_1 + A_n\right)}{2}$$

Hence, the theorem is proved.

Corollary 1. $S_n = \dfrac{n\left(2A_1 + (n-1)d\right)}{2}$

Proof: The corollary follows from Theorem 1 by using

$\mathbf{A_n = A_1 + (n - 1)d}$

Corollary 2. The sum of arithmetic sequence (T_n) 1,2,3,4, 5, ….n is given by $T_n = \frac{n(n+1)}{2}$

Proof: The corollary easily follows by Theorem 1 with $A_1=1$ and $A_n = n$.

Problem 7

The first term of an arithmetic sequence is equal to 10 and the common difference is equal to 5. Find the sum of 50th term

Solution. Using $A_n = A_1 + (n - 1)d$, we have

$$A_{50} = 10 + (50 - 10)5$$

$$= 210$$

Now $S_{50} = \dfrac{50\left(10 + 210\right)}{2}$

$=5550$

Therefore, the sum of 50^{th} term is 5550.

Problem 8.

Suppose that you play black jack at Las Vegas on June 1 and lose $1,000. Tomorrow you bet and lose $15 less. Each day you lose $15 less that your previous loss. What will your total losses be for the 30 days of June?

Solution

This is an arithmetic series problem. Note that

$A_1 = 1000$ and $d = -15$

We can calculate

$A_{30} = 1000 - 15(30 - 1) = 565$

Now we use the formula

$S_{30} = 30/2\ (1000 + 565) = 23{,}475$

You will lose a total of $23,475 during June.

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Theorem 6: Then partial sum of arithmetic sequence $1, 2, 3, 4 \ldots (2^k - 1)$. is a triangular number.

Proof: Let T be the sum of this arithmetic sequence. Note that:

$$T = 1+2+3+4\ ...+(2^k-1)$$
$$= \frac{\left(2^k-1+1\right)\left(2\ \ -1\right)}{2}$$
$$= \frac{2^k\left(2\ \ -1\right)}{2}$$
$$= 2^{k-1}\left(2^k-1\right) = \frac{m(m+1)}{2},\ where\ m = \left(2^k-1\right).$$

Hence, T is a triangular number.

References:

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