THINKING ON THE THREE GENERATIONS OF PROSPECT THEORY

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Synopsis:

In this article, we review and make some comments on the research of behavioral finance based on three generations of prospect theory by China's scholars. We discuss their contributions from six aspects including the difference of the values of parameters of value function and weight function; the difference between individual's decision behavior and group's decision behavior; the explanation of insurance behavior from perspective of prospect theory; and other important factors.
Thinking on three generations of prospect theory

Abstract
In this article, we review and make some comments on the research of behavioral finance based on three generations of prospect theory by China's scholars. We discuss their contributions from six aspects including the difference of the values of parameters of value function and weight function; the difference between individual's decision behavior and group's decision behavior; the explanation of insurance behavior from perspective of prospect theory; the multinational comparison of risk attitudes of firms; the consistency of the expected utility theory and the prospect theory; and the effect of cultural and gender difference on the prospect theory.

Keywords: Prospect theory; review; research of behavioral finance in China
Introduction

The value function based on prospect theory (Kahneman and Tversky, 1979) shows that the decision maker is risk averse to the gains, but he (she) is risk seeking to the loss, that is, the curve of value function generally is concave for gains and commonly convex for losses. And also the decision maker is more sensitive to the loss than to the benefit. In other words, the curve of value function is steeper for losses than for gains. While this is commonly considered in the context of investment, real or financial, Rychalski and Hudson, 2017, note similar behaviors as well in context of consumer satisfaction.

A new version of prospect theory: Cumulative prospect theory (Tversky, 1992) uses new experiment to confirm fourfold pattern of risk aversion: a. Risk aversion for gain of high probability; b. Risk-seeking for losses of high probability; c. Risk-seeking for gains of low probability; d. Risk aversion for losses of low probability. They also get the following conclusions: there is a nonlinear transformation of the probability scale, which over-weights small probabilities and under-weights moderate and high probabilities. The exponents of value function and weighting function are generally less than 1.

Barberis (2013) summarizes four elements of prospect theory: 1) reference dependence; 2) loss aversion; 3) diminishing sensitivity; 4) probability weighting.

Subsequent to Tversky and Kahneiman (1992) paper on cumulative prospect theory, several studies have used more sophisticated techniques in conjunction with new experimental data to estimate the value function $v(\cdot)$ and the weighting function $w(\cdot)$ more accurately (Gonzalez and Wu 1999, Abolellaoui 2000, Bruhin, Fehr-Duda, et al., 2010). These studies confirm the properties of these function identified by
Kahneman and Tversky: the loss aversion and diminishing sensitivity of features of the value function, and the inverse S-shape of the weighting function. They provide especially strong support for probability weighting (Barberis, 2013).

Schmidt, Starmer and Robert (2008) present a new theory of decision under uncertainty: third-generation prospect theory. It retains the predictive power of previous version of prospect theory, but extends that theory by allowing reference points to be uncertain while decision weights are specified in a rank-dependent way.

They point out that first and second generation prospect theory have a common limitation: the reference points from which prospects are evaluated are assumed to be certain. However, they have set themselves the task of generalizing cumulative prospect theory so that it can encompass uncertain reference points.

Besides above review, there is still a lot of other approaches from all over the world (For the review of the literature, please see Barberis, 2013). It also attracts the eyes of many China's authors. There are several papers in Chinese literature to study behavioral economics or behavioral finance based on prospect theory. In this article, we will make some discussion based on some literature in Chinese and the view points of ourselves.

1. Introduction of fundamental models of three generations of prospect theory

In prospect theory, Kahneman and Tversky propose value function which can be

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1 The original version (or first generation)(Kahneman and Tversky, 1979; and cumulative prospect theory (or the second -generation) is featuring rank-dependent decision weights (e.g. Starmer and Sugden, 1989; Luce and Fishburn, 1991; Tversky and Kahneman, 1992; Wakker and Tversky, 1993.)
expressed as follows:

\[ v(\Delta x) = \begin{cases} 
(\Delta x)^\alpha & \Delta x \geq 0 \\
-\theta(-\Delta x)^\beta & \Delta x < 0 
\end{cases} \]

where \( \alpha, \beta \) are the coefficients of risk attitude, and \( 0 < \alpha, \beta < 1 \). The greater the values of \( \alpha, \beta \), the more risk seeking the decision maker is. \( \theta \) is the coefficient of loss aversion. If \( \theta > 1 \), it shows that the decision maker is more sensitive to the loss than to the benefit.

Tversky and Kahneman in 1992 propose the expression of probability weight function which can be expressed as:

\[ \pi(p) = \begin{cases} 
\frac{p^\gamma}{(p^\gamma + (1-p)^\gamma)^{1/\gamma}} & \Delta x \geq 0 \\
\frac{p^\delta}{(p^\delta + (1-p)^\delta)^{1/\delta}} & \Delta x < 0 
\end{cases} \]

where \( \gamma, \delta \) is less than 1, that is weight is greater than probability with small probability and weight is less than probability with middle and large probability.

Schmidt, Starmer and Sugden (2008) propose the third generation prospect theory. They impose the restriction that the relative value function takes the form:

\[ v(f(s_i), h(s_i)) = u(z), \text{ where } z = f(s_i) - h(s_i), \text{ } u(\cdot) \text{ is the counterpart of the value function in earlier generations of prospect theory, and the relative value function can be interpreted as the desirability of the consequence of act } f \text{ in state } s_i \text{ relative to the consequence of a reference act } h \text{ in the same state.} \]

They let \( V(f, h) = \sum_i v(f(s_i), h(s_i))W(s_i, f, h) \) be the expectation of relative value function, where \( v(f(s_i), h(s_i)) = u(z) \), is the decision weight assigned to state \( s_i \).
when \( f \) is being evaluated from \( h \). This function is strictly increasing in its first argument, with \( f(s_i) - h(s_i) = 0 \), when \( f(s_i) = h(s_i) \).

They define cumulative decision weights as follows:

\[
W(s, f, h) = \begin{cases}
    w^+(\pi_i) & \text{if } i = m^+ \\
    w^+(\Sigma_{j \geq i} \pi_j) - w^+(\Sigma_{j > i} \pi_j) & \text{if } 1 \leq i \leq m^+ - 1 \\
    w^-((\Sigma_{j \geq i} \pi_j) - w^-((\Sigma_{j > i} \pi_j) & \text{if } -m^- + 1 \leq i \leq -1 \\
    w^-((\pi_i) & \text{if } i = -m^-
\end{cases}
\]

where the state with weak gains are indexed \( m^+, \ldots, 1 \) and the states with strict losses are indexed \( -1, \ldots, -m^- \).

Specify a power function for \( u(\cdot) \) as follows:

\[
u(z) = \begin{cases}
z^\alpha & \text{if } z \geq 0 \\
-\lambda|z|^\alpha & \text{if } z < 0,
\end{cases}
\]

where the parameters \( \alpha \) and \( \lambda \) are required to be strictly positive. \( \alpha \) controls the curvature of the value function and \( \lambda \) controls attitudes to gain and loss. If \( \alpha < 1 \), this function is concave in the domain of gains and convex in the domain of losses. With \( \lambda = 1 \) there is loss neutrality. For \( \lambda > 1 \), there is loss aversion: losses are weighted more heavily than gains and vice versa.

They establish model of decision weights by means of a single-parameter probability weighting function. For reasons of parsimony they impose the restriction of identical weighting functions for gains and losses (\( w^+(\pi) = w^-(\pi) \)). Hence for the purpose of parameterization the probability weighting function is denoted simply by \( w(\pi) \); it takes the form
\[ w(\pi) = \frac{\pi^\beta}{\left(\pi^\beta + (1-\pi)^\beta\right)^\frac{1}{\beta}}, \text{ with } \beta > 0, \text{ where } \pi \text{ is state probability.} \]

2. Main research on behavior economics and behavior finance based on perspective theory by Chinese scholars

2.1 About the values of parameters of value function and weighting function

Since all of above conclusions are based on the experiment in U.S. It is doubtful if it has general meaning. In fact, the results of experiment by an author named Jianming Zhen (2007) in China shows the parameters for value function and the function of probability weights are quite different (please see Table 1).

**Table 1. Function parameters from American and Chinese experiment**

<table>
<thead>
<tr>
<th></th>
<th>$\alpha$</th>
<th>$\beta$</th>
<th>$\gamma$</th>
<th>$\delta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>America</td>
<td>0.88</td>
<td>0.88</td>
<td>0.61</td>
<td>0.69</td>
</tr>
<tr>
<td>China</td>
<td>1.21</td>
<td>1.02</td>
<td>0.55</td>
<td>0.49</td>
</tr>
</tbody>
</table>

The experiment results by Tversky and Kahneman show that $\alpha, \beta < 1$, so the sensitivity to gains and losses decreases gradually, but those by Zhen in China show that $\alpha, \beta > 1$, so the sensitivity to gains and losses increases gradually. Figure 1 are the comparison of value function based on the experiments on Chinese and American.

We think that the reason that results in the difference of parameter values between America and China is the difference of risk attitudes of decision makers to gains and losses between America and China.
Zheng (2007) points out that the difference of risk attitude among different countries probably comes from different traditional life and idea. China is a large country of agriculture. Most of population is agriculture population. The life of people is very stable and people rarely needs to face the problem of losses. However, the people who engages business activity possibly have more chance to face the problem of losses. Therefore, as long as facing the problem of losses, they are very unwillingly accepting them. They intensely have a kind of psychology to make a bet and especially tend to select risk so as to make CE (certainty equivalent) seriously diverge from EV (expected value). So it is worth to study the effect of different culture background on the risk attitudes of the decision makers.

Li (1995) investigates by experiment using the samples from Australia and China and summarizes that in order for $\pi(p)$ to obey the maximization principle, they are compelled to derive from the experiments that $\pi(p) < p$ for very small $p$ or $\pi(p) > p$ for very large value of $p$. Just opposite to Cumulative Theory, where the
authors points out that $\pi(p) > p$ for very small $p$ but $\pi(p) < p$ for very large value of $p$. Li suggests: "The present results are not at all surprising if the maximization hypothesis itself is deemed to be false. The conclusion drawn from this study is that there is no such thing as the $\pi$ function and that the $\pi$ function was postulated by prospect theory only because no consideration was given to the possibility of discarding the maximization principle. Though expected utility theory is known to be wrong through the Allais paradox, to rescue it by proposing the $\pi$ function is to do nothing but hide an old mistake under a new one".

2.2 About the difference between individual's decision behavior and group's decision behavior.

Wen, Rao and Yang (2010) take a whole stock market as an entire entity, use the flow of information extracted by EGARCH Model as the proxy variable of change in wealth, and then use a two stage power function as the representation of the value function to study the daily return data from stock markets of 10 countries or regions. Their empirical results show that the value function of all the 10 stock markets presents the shape of inverse-S, instead of the S-shape of value function generated by Tversky and Kahneman (1979). The 10 countries are France(CAC40), German(DAX), England(FSI), USA(S&P500), Switzerland(CSMI), China(SSE), Japan(N225), Austria(ATX), Netherland(AEX ), and Italy (MIBTEL).

Table 2 lists the values of parameters of $\alpha$ and $\beta$ of value functions using the data of 10 countries from 2000 to 2009 based on EGARCH Model.
### Table 2 The estimate of parameters of $\alpha$ and $\beta$ of ten countries by empirical method

<table>
<thead>
<tr>
<th>Names of stocks</th>
<th>$\alpha$</th>
<th>$\beta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>S&amp;P500</td>
<td>1.403263</td>
<td>1.079569</td>
</tr>
<tr>
<td>EST</td>
<td>1.464533</td>
<td>1.219373</td>
</tr>
<tr>
<td>SMI</td>
<td>1.531092</td>
<td>1.228296</td>
</tr>
<tr>
<td>CAC40</td>
<td>1.497415</td>
<td>1.234854</td>
</tr>
<tr>
<td>DAX</td>
<td>1.514574</td>
<td>1.156375</td>
</tr>
<tr>
<td>SSE</td>
<td>1.079661</td>
<td>1.158955</td>
</tr>
<tr>
<td>N225</td>
<td>1.151637</td>
<td>1.067991</td>
</tr>
<tr>
<td>AEX</td>
<td>1.681457</td>
<td>1.377977</td>
</tr>
<tr>
<td>MIBTEL</td>
<td>1.452415</td>
<td>1.164320</td>
</tr>
<tr>
<td>ATX</td>
<td>1.177498</td>
<td>1.284861</td>
</tr>
</tbody>
</table>

The rise of the price in stock market does not restrain but expand the need to the stock and make the stock price rise further. However, the decline of the price of security would decrease the need to this security and results in the further decline of the price of this security. Therefore, the investor who is aware this point will display the characteristic of pursuing risen up and abandon got down. It inevitably promotes the stock price to extremes, causes exceeding reaction of stock price, and produces stock market foam at last. Therefore, it is worth to further study the difference between individual decision behavior and group's decision behavior.
2.3 About insurance (Please also see Liu, Zhang and Wang, 2006)

For property which exists stochastic losses, not buying insurance means to bear stochastic losses, but buying insurance means to bear certain losses (pay premium). Based on prospect theory, people is risk averse to return, that is to say that they prefer certain gains to uncertain gains. However, people is risk seeking to the losses, that is to say that they prefer uncertain losses to certain losses. Therefore, the curve of value function is S shaped.

However, in insurance, this is not true. The insurance buyer is risk seeking when facing gains, that is to say, they prefer big uncertain claim benefit to certain premium saved, but he is risk averse when facing losses, that is to say, the high losses with low probability is worse than the subjective feeling on losing certain and small amount of premium. Therefore, the value function for insurance is inverse S-shaped (Please see Figure 2).

For example, for participating life insurance and investment linked insurance, they exist
the characteristic of gains. The person who buys life insurance can get the uncertain benefits but the person who does not buy life insurance can save the premium which can be considered as certain gain. Therefore, the person who buy participating life insurance or investment linked insurance is risk seeking to gains.

2.4. The multinational comparison of risk attitudes of firms

Zheng (2009) compares the risk attitudes between China's firms and American-European firms by using of prospect theory. They found that: (1) China's firms appeared to be stronger risk propensity than American-European firms no matter whether the return is above or below the target value; (2) Contrasting to American firms, China's firms' risk-return relationship was negative in whole economic cycle; (3) China's firms' risk attitudes is not completely consistent with prediction of prospect theory.

His research shows that the basic cause resulting in the negative correlation between risk and return of cross enterprise is from the interior of the firms. The environment factors can only change the degree of negative correlation but cannot change the direction of the correlation. The final results of the effect of environment factors are dependent on the quality of factors from the interior of the enterprises. Therefore, the research on the correlation between risk and return of cross enterprises should be focus on the difference of main factors affecting risk and return in the interior of the enterprises.

Zhen points out that the risk attitudes affecting manager should be a more complicated system. The current research on this problem, no matter it is based on expected utility theory or prospect theory, appears oversimplified. Kahneman and Tversky (1979) do
not consider the behavioral motive and behavioral selection under different behavioral roles so as to result in its explanation ability limited, although they point out the un-stability of risk preference. For the decision making under uncertainty, the important thing is not losses or gains but who will gain or who will loss and what kind of sharing responsibility or distribution it has. Therefore, It should introduce the systemic factors for modification in application of prospect theory.

In fact, Zheng carries out his analysis using two different time period data: 1996-2000 and 2001-2005 respectively. The results show that the degrees of negative correlation between risk and return in 1996-2000 are all higher than those in 2001-2005. He found by connecting with the operation state of national economy that during 1996-2000, the national economy was being in the string stage of economic cycle, but during 2001-2005, it was being in the expansion stage of economic cycle. It illustrates that the risk attitude of firms and the variation of firms' performance is connected with the environment of macro-economy of a country.

Zheng makes a comparison with the results of Fiegenbaum and Thomas (1986). He found that the correlation between risk and return of America firms is not stable, that is, in different observation periods, the direction of correlation is different (In 60's, the risk and return of firms are positively correlated, but in 70's, the risk and return of firms are negatively correlated). However, the correlation between risk and return of China' firms is relatively stable and what is different is only the difference of degree of correlation but the correlation direction is not changed. Zhen points out that (1) the economic environment of China has higher uncertainty and the firm faces greater
opportunity of success and threat of failure; (2) The firms of China are more willing to take risks. It demonstrates that the behavior of China's firms does not completely conform to prospect theory; (3) The problem of risk control of China's firms is greater than developed countries.

Hsee and Weber (1999) compared the risk attitudes of China's firms and those of America's. They found that China's firms is more risk seeking than those of America's. They proposed "cushion hypothesis" to explain their discovery. They think that different culture background results in this phenomenon. However, we think that the reason why the China's firms more likely take risk in investment cannot be explained in that China's society encourages people to take risk. In fact, Chinese in other fields such as academic research and health care is more conservative. The only reason is that Chinese generally has the attitude to urgently become rich. This kind of rash attitude becomes the blinded and unintended, that is, greatness, eager for quick success.

Zheng thinks that firstly, the economy of China is in the period of high speed increase. Since 1978, China has realized high economic growth nearly for 30 years. It brings great amount business opportunity and makes it possible for people to obtain high return by taking risks. Therefore, the managers of firms appears intense risk seeking attitudes.

Secondly, China is in the transition stage from planning economy to market economy. When the old system is not broken and new system is not established or improved, the people' s behavior is frequently in the state of no constraint or constraint slack. The problem of controlling insiders still does not solved effectively. It is expected to establish the market of managing agent and blacklist. There exists a big acting space
that officials collude with merchants. In this situation, the managers naturally have great opportunities to construct empire-preservation and entrenchment. In the stimulating of high speed increase, investment hungry and thirsty would appear more intensive, such as: hotness of real estate. It is more rampant for the corruption behavior under the banner of adventure. Thirdly, there exists the misunderstanding that focus on superstition or crushing on huge enterprises. Since the middle of 90's, the enterprises in China began to perform diversification in large scale. It becomes the topic of people relishing to create aircraft carrier of industry. The enthusiasm for diversification management did not diminish, although Asia financial storm gives Korea who is keen to establish huge enterprises a great hitting. To instead, they quicken their pace to diversity in entering 21centry. It forms obvious contradiction to that the developed countries are trending towards refocusing one after another.

2.5. About the consistency of the expected utility theory and the prospect theory

Bian and Cai (2005) approach the consistency of the expected utility theory and the prospect theory. They present a proof that under the axiom of the algebra of combination, these two theories are exactly identical.

Kahneman and Tversky (1979) point out that their experiments show that the axiom of the algebra of combination is not set up. However, Bian and Cai (2005) think that Kahneman and Tversky (1979)'s experiment is the decision of two stages and what the subjects make is middle decision. Two stage decision is different from one stage decision. And the axiom of the algebra of combination is only suitable to one stage decision. Therefore, it cannot illustrate that the axiom of the algebra of combination is
not set up. Bian and Cai accept the axiom of the algebra of combination in their article. Based on this axiom, they prove that the weight is equal to corresponding probability, that is, $\pi(\alpha) = \alpha$.

Following is their proof.

The axiom of the algebra of combination can be expressed by the following mathematical formula:

$$(1 - \gamma)[(1 - \alpha)u_0 + \alpha v_0] + \gamma[(1 - \beta)u_0 + \beta v_0]$$

$$= [(1 - \gamma)(1 - \alpha) + \gamma(1 - \beta)]u_0 + [(1 - \gamma)\alpha + \beta]v_0$$

(1)

The right side of equation (1) expresses such kind of prospect, that is, $u_0$ is obtained in the probability of $(1 - \gamma)(1 - \alpha) + \gamma(1 - \beta)$ and $v_0$ is obtained in the probability of $\gamma$.

The left side of equation (1) expresses such kind of prospect, that is, $A$ is obtained in probability of $1 - \gamma$ and $B$ is obtained in probability of $\gamma$, where $A$ expresses that $u_0$ is obtained in probability of $1 - \alpha$ and $v_0$ is obtained in probability of $\alpha$, $B$ expresses that $u_0$ is obtained in probability of $1 - \beta$ and $v_0$ is obtained in probability of $\beta$.

The results they want to prove is that for a risk asset which obtains $u$ in probability of $(1 - \alpha)$ and obtains $v$ in probability of $\alpha$, that is, $w \sim (1 - \alpha)u + \alpha v$. If satisfying with the axiom of the algebra of combination, the decision weighting function in prospect theory $\pi(\alpha)$ is identically reflected, that is, $\pi(\alpha) = \alpha$.

Proof: For first situation, that is, $V(px + qy) = \pi(p)v(x) + \pi(q)v(y)$, when $p + q < 1$ or $x \geq 0 \geq y$ or $x \leq 0 \leq y$.

$$\pi(\alpha) = \alpha. \quad (2)$$

When $x$ is determined asset, $V(x) = v(x)$. $\pi$ function is called as decision weighting function. It expresses the subjective evaluation to objective probability and it reflects
the effect of probability on prospect value.

The utility of \( w \) can be written as:

\[
f(w) = \pi(1-\alpha)f(u) + \pi(\alpha)f(v), \text{ then } \pi(\alpha) = \alpha. \tag{3}
\]

Given \( u_0 \) and \( v_0 \), and assuming \( f(u_0) = 0, f(v_0) = 1 \), then for any values of \( \alpha \) and \( \beta \ (0 \leq \alpha < \beta \leq 1) \), let \( u_i = (1-\alpha)u_0 + \alpha v_0, v_i = (1-\beta)u_0 + \beta v_0 \). From equation (3), it can be obtained that

\[
f(u_i) = \pi(1-\alpha)f(u_0) + \pi(\alpha)f(v_0) = \pi(\alpha), \tag{4}
\]

\[
f(v_i) = \pi(1-\beta)f(u_0) + \pi(\beta)f(v_0) = \pi(\beta). \tag{5}
\]

For any values of \( \gamma \ (0 \leq \gamma \leq 1) \), let \( w = (1-\gamma)u_i + \gamma v_i \). From equation (3) and using equations of (4) and (5), it can be obtained that

\[
f(w) = \pi(1-\gamma)f(u_i) + \pi(\gamma)f(v_i)
= \pi(1-\gamma)\pi(\alpha) + \pi(\gamma)\pi(\beta) \tag{6}
\]

From the definitions of \( u_i \) and \( v_i \), and also from the axiom of algebra of combination, it can be obtained that

\[
w = (1-\gamma)[(1-\alpha)u_0 + \alpha v_0] \\
+ \gamma[(1-\beta)u_0 + \beta v_0] \tag{7}
= [(1-\gamma)(1-\alpha) + \gamma(1-\beta)]u_0 \\
+ [(1-\gamma)\alpha + \gamma\beta]v_0
\]

From equation (3), it can be obtained that

\[
f(w) = \pi[(1-\gamma)(1-\alpha) + \gamma(1-\beta)]f(u_0) \\
+ \pi[(1-\gamma)\alpha + \gamma\beta]f(v_0) \tag{8}
\]

By combining equations of (6) and (8), and for any values of \( \alpha, \beta, \gamma \in [0,1] \), it can be found that

\[
\pi[(1-\gamma)\alpha + \gamma\beta] = \pi(1-\gamma)\pi(\alpha) + \pi(\gamma)\pi(\beta) \tag{9}
\]

Based on prospect theory, \( \pi(0) = 0, \pi(1) = 1 \). Letting \( \alpha = 0 \) in equation (9), it can be obtained that

\[
\pi(\gamma\beta) = \pi(\gamma)\pi(\beta) \tag{10}
\]
Therefore, there exist a real number which makes \( \pi(\alpha) = \alpha^k \) \hspace{1cm} (11)

For the proof of equation (11), please see appendix.

Putting equation (11) into equation (9), obtain that

\[
[(1 - \gamma)\alpha + \gamma\beta]^k = (1 - \gamma)^k \alpha^k + \gamma^k \beta^k
\]

(12)

Letting \( \gamma = \frac{\alpha}{\alpha + \beta} \), obtain that

\[
\left( \frac{2\alpha\beta}{\alpha + \beta} \right)^k = 2 \frac{\alpha^k \beta^k}{(\alpha + \beta)^k}
\]

(13)

From equation (13), we know that \( 2^k = 2 \). Therefore, \( k = 1 \) and \( \pi(\alpha) = \alpha \) \hspace{1cm} (14)

For the second situation, that is, \( p + q = 1 \) and \( x > y > 0 \) or \( x < y < 0 \),

\[
V(px + qy) = v(y) + \pi(p)[v(x) - v(y)]
\]

(15)

Since the nature of decision weighting function is obtained by inducting the normal situation of equation (2). Therefore, Bian and Cai (2005) only make proof based on equation (2).

To sum up, whether the sum of decision weights is equal to 1 or not, as long as satisfying with the axiom of the algebra of combination, the decision weight is certainly equal to objective probability. Therefore, in this situation, the established model for prospect theory is consistent with expected utility function.

2.6. About the effect of cultural and gender difference on the prospect theory

Jin, Jiang and Wang (2014) explore the features of risk decision in the context of Chinese culture by experiment. There are 400 Chinese undergraduates are participating in this experiment, where there are 200 male students and 200 female students. The information of these students is coming from internal web of Zhejiang University. The results show that under the condition of profit, the Chinese respondents showed the
same preference reversal as the Westerners. However, in the case of loss, Chinese and Western participants show different behavior feature, that is, Chinese respondents do not show obvious preference reversal. Further analysis reveals that the Chinese male and female show the opposite choice preference under the condition of low-probability loss. Chinese male prefers the choice with lower risk and larger amount of loss. However, Chinese female prefers the choice with higher risk and smaller amount of loss. This results shows that the individual of risk decision making with different cultural backgrounds have different risk preferences, and the gender has an impact on the risk preference.

Jin, Jiang and Wang (2014) point out that the research before discovers that Chinese is more risk-loving (Weber and Hsee, 1998, Hofstede, 2005). Hofstede proposes "Buffering hypothesis" to explain the reason why Chinese is more risk-loving than Westerners. He thinks that in the countries of collectivism such as China, the family or other members in the group will try their every effort to help anyone in the group if he faces tremendous or even catastrophic risks. However, in the countries of individualism such as America, the individuals will choose to be responsible for himself. Therefore, the people in the society of collectivism will see collectivism as the buffering which offsets possible losses. Weber and Hsee think that the degree of risks Chinese perceives would be less than those perceived by American because the risks are reduced objectively due to the buffering effect of negative results. Fehr-Ducla, Gennaro and Schubert (2006) discusse the relationship between gender and risk bearing. The experiment shows that different gender has different function of weighting. However,
there is no obvious difference in value function for different gender. the sensitivity of female to the change of probability is lower than that of male. At the same time, female is more tense to under-estimate big probability of earnings. Female is more pessimistic to earnings, meanwhile, they are more risk averse than male does. Contrasting to male, female appears more risk averse corresponding to earning area with larger probability and loss area with small probability.

Different from above research, Jin, Jiang and Wang (2014) discovers that Chinese, especially Chinese male is more risk averse in the condition of loss. The one reason may be that the study before did not consider the problem of culture difference under the special background that considers extremely small probability in the condition of losses. The another reason may be the problem of deep-rooted "face" that Chinese male has. "Face" has become one of important characteristics of interpersonal communication between Chinese due to Chinese traditional culture and Confucius philosophy. By fundamental speaking, "face" is a kind of self-construction. In such kind of counties with the culture of collectivism as China, individual is not a complete entity. For example, the male in China would see themselves as sons, brothers, husbands and fathers, but absolutely not only as themselves. Therefore, it is possible that the male will more likely does not tolerate failure than the female does, because the decision making will affect the family members and other persons who has benefit connection with the decision maker. In the condition of losses, the Chinese male bears more responsibilities which come from society and their families. And since they have a stronger self-esteem, it results in the fact that Chinese male more likely does not willing
to tolerate failure.

2.7. Other researches

2.7.1. About prospect theory, liquidity constraints and asymmetric consumption

Based on the prospect theory advocated by behavioral economics, Kong (2005) researches the consumption behavior of Chinese urban households. The results show that if taking the expectation of income change as the benchmark, the change of real income can split into two phases which are better than expectation and worse than expectation respectively. With the characteristic of loss aversion discussed by prospect theory, the consumption will be asymmetric in different phases. He sets up a model to test this hypothesis by applying GMM. Results support the prospect theory and reject presumptions of myopia and liquidity constraints. Finally, he tests the time varying factor, and finds out that the systemic transition has effects on consumption in a certain extent.

2.8.2 About the optimal asset allocation based on dynamic loss aversion

Jin, Wang and Gao (2014) establish a model of portfolio optimization with dynamic loss aversion by considering the psychological characteristics of loss aversion from the perspective of behavioral finance. They develop the research of Fortin and Hlouskova (2011) by using the value of asset portfolio to instead of the rate of return of asset portfolio. Dividing China's stock market into three states including rise, decline and consolidation, they empirically study the optimal asset allocation and the performance of the model with dynamic loss aversion. They also compare their model with the model of static loss aversion portfolio as well as mean-variance and CVaR portfolio models.
Finally, they carry out the robust test on dynamic model under the condition of considering trade cost. The results show that the investors with dynamic loss aversion have different ratio of optimal asset allocation under different market conditions. Meanwhile, the model of dynamic loss aversion clearly outperforms static model, the optimal portfolio model based on mean-variance criterion and the optimal portfolio model based on CVaR criterion. 2

2.7.3 About group grey target\(^2\) decision

Yan and Liu (2014) propose a method of group grey target decision making based on prospect theory by considering that all decision makers have impact on group decision making based on the fact that every decision maker has expected grey target on every index. They define a prospect value function by using the expect grey target as reference point. They use a linear operator with the features of "rewarding good and punishing bad" to standardize the prospect value. The positive or negative sign of the standardized values can reflect adequately whether the values of indices hit the target. They also propose the weighting model of decision maker based on group's consistency and influence of the maximum and minimum evaluation on the deviation of decision. And whether the values of indices hit the target is judged according to the positive or negative sign of comprehensive prospect values. Finally, they illustrate the feasibility and effectiveness of the proposed model by the selection of the emergency plan.

2.7.4 About weighting risk and uncertainty

\(^2\) The name of "grey target" comes from grey system theory which is developed by Prof. Deng Julong in 1980s(Please see Deng, 2002). Grey system theory has classified its research objectives into three kinds of 'black', 'white' and 'grey' according to some cognitive hierarchy. 'black' means totally unknown; 'white' means totally known and 'grey' means partly known and partly unknown.
Zhou and Wang (2005) proposed a three-stage framing approach by allowing the probability weighting function to depend on the type of uncertainty and taking different processing mode on gain and loss so that it makes decision maker obtain more accurate decision weights.

**Conclusions**

In this article, we review and make some comments on the research on prospect theory and cumulative prospect theory by China's researchers. We discuss their studies mainly from six perspectives including the difference of the values of parameters of value function and weight function, the difference between individual's decision behavior and group's decision behavior, the explanation of insurance behavior from perspective of prospect theory, the multinational comparison of risk attitudes of firms', the consistency of the expected utility theory and the prospect theory, and the effect of cultural and gender difference on the prospect theory.

Although more than thirty years has passed since Kahneman and Tversky propose prospect theory, the main research of China's scholars on it only has about ten year. And we think that the most important finding would be those studying the culture and gender difference on the risk attitudes, the difference of the values of the parameters of value function and weighting functions, and the difference of risk attitude of firm's among China and other countries. The male in China more likely does not willing to tolerate failure because of their stronger self-esteem, so they are more conservative in decision making. This result is quite different from the general thinking that male is usually more
risk seeking. China's firms appeared to be stronger risk propensity than American-European firms no matter whether the return is above or below the target value, and China's firms' risk attitudes is not completely consistent with prediction of prospect theory.

Although China's researchers have developed prospect theory and cumulative prospective theory from several aspects, it is very limited comparing with the researches from other countries, especially from Western counties. Even so, we still hope the achievements of China's researchers on the study of prospect theory can be known and perhaps to be acknowledged further if their researches are valuable.

Reference

Abdellaoui, M., 2000. Parameter-free elicitation of utility and probability weighting function, Management Science. 46, 1497-1510.


**Appendix**

The proof of equation (11)

For any values of $x$ and $y \in [0,1]$, if function of $f(x)$ satisfies with

$$f(x, y) = f(x)f(y)$$

(16)

and also $f(x)$ is not identically equal to 0, then $f(x)$ is power function. There exists a real number which makes $f(x) = x^k$  

(17)

Proof: Letting $F(x) = f(e^x)$, then

$$F(x + y) = f(e^{x+y}) = f(e^x e^y) = f(e^x)f(e^y) = F(x)F(y)$$

(18)

Therefore, $F(x) = F\left(\frac{x}{2} + \frac{x}{2}\right) = \left(F\left(\frac{x}{2}\right)\right)^2 \geq 0$

(19)

Since $F(x)$ is not identically equal to 0, there exists $x_0$ which makes $F(x_0) > 0$ established. Therefore, $F(x_0 + 0) = F(x_0)F(0)$, that is, $F(0) = 1$.

From equation (18), we know that for any values of $m$ and $n$. It can be obtained that
Therefore, \[ F\left(\frac{x}{n}\right) = \left(F(x)\right)^{\frac{1}{n}}, \quad F\left(\frac{m}{n} \cdot x\right) = \left(F(x)\right)^{\frac{m}{n}} \]

(20)

For any real numbers of \(c\), there exists rational number series \(p_j \to c(j \to \infty)\).

From equation (20), it can be obtained that \( F(p_j x) = \left(F(x)\right)^{p_j} \).

Letting \(j \to \infty\), it is obtained that \( F(cx) = \left(F(x)\right)^{c} \).

Letting \(x = 1\), it is obtained \( F(c) = \left(F(1)\right)^{c} \), that is, \( F(x) = \left(F(1)\right)^{x} \).

Letting \(F(1) = a\), it is obtained that \( f(e^x) = F(x) = a^x \).

Letting \(y = e^x\) then \( f(y) = a^{\ln y} = (e^{\ln a})^{\ln y} = y^{\ln a} \).

Letting \(k = \ln a\), then \( f(x) = x^k \).