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CALCULATION OF CENTROIDS AND CENTER OF MASS USING MATLAB

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Abstract: The centroid or a center of gravity of any object is the point within that object from which the force of gravity appears to act. It is important for building bridges, dams and roofs of buildings which can be semi-elliptical or semi-circular. Let the coordinates of the centroid be $\bar{x}$ and $\bar{y}$ in x, y direction. The calculation of these coordinates is done by mathematical integration given by the formulas $\bar{x} = \frac{\int x dA}{A}$ and $\bar{y} = \frac{\int y dA}{A}$ where A is the area of the object. In this project we will utilize MATLAB with its symbolic toolbox to do this mathematical integration for areas such as quarter and semicircular area, quarter and semielliptical area, parabolic spandrel and semi-parabolic area. In the beginning of this project a simple calculation is demonstrated without using the software.

Key Words: centroid, center of mass, distributed forces

Introduction

The purpose of this project is to understand the concept of distributed forces acting on as body or on an object and the necessity of calculating its centroid or center of mass. For example, consider the design of high-performance sail-boats where both air-pressure distributions on the sails and water-pressure distributions on the hull must be taken into account. Another example is huge dams where they are subjected to three different kinds of distributed forces such as the weights of their constituent elements, the pressure forces exerted by the water of their submerged face and the pressure forces exerted by the ground on their base. A precast section of roadway for a new interchange on interstates is an additional example where the location of the centroid and the behavior of the roadway under loading are established. Finally, the roofs of the buildings must be able to support not only the total weight of the snow but also the non-symmetric distributed loads resulting from drifting of the snow. A typical example of a three-dimensional body is the modified Boeing 747 transporting a space shuttle where the flight characteristics are predicted by determining the center of gravity of each craft. However, this is beyond the scope of this project.

The aforementioned examples demonstrate how important is to know the location of the centroid for a civil engineer. Therefore, the main purpose of this project is to utilize the MATLAB software to locate the centroid of common shapes such as quarter and semicircular, quarter and semielliptical, parabolic spandrel and semi-parabolic. In doing so, the knowledge of calculus especially integration is necessary. These integrals are very difficult to be evaluated. We will be writing simple MATLAB code to do the integration for us and the results are provided. Our variables are algebraic and the reader can substitute any specific numerical values and the specific centroid. A composite plate is also demonstrated without the use of integration. We encourage the reader to do examples by hand and learn the technique before utilizing the software. A very good collection of MATLAB programs in engineering statics is by Shah, et.al. [1]. The Mathematical background of this work is given in Salas, et.al. [2], a good book regarding MATLAB is by Palm [3] and a good book for engineering Statics is by Beer et. al, [4].

Evaluating the Centroid of common shapes

Before proceeding with MATLAB examples, an example of finding the centroid of a trapezoidal area is shown below in figure 1.

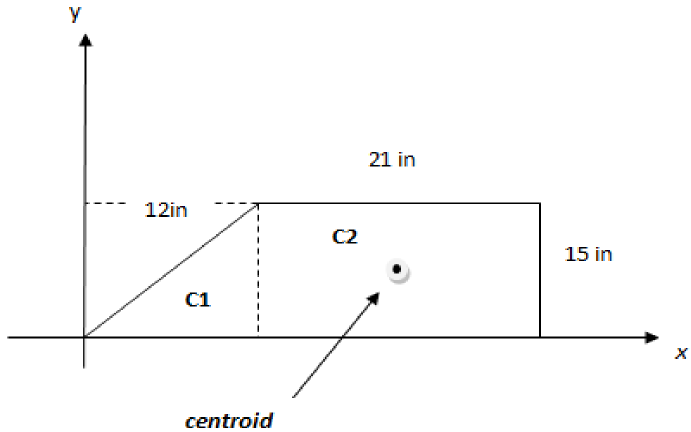

Figure1. Centroid of a Trapezoidal area.

This region consists of a triangle (c1) and a rectangle (c2). So, we find the areas and centroids of each region. The following table is formed.

	A	$\bar{x}$	$\bar{y}$	$\bar{x}$ A	$\bar{y}$ A	$\sum A$	$\sum \bar{x}A$	$\sum \bar{y}A$
C1	90	8	5	720	450	405	7807.5	2812.5
C2	315	22.5	7.5	7087.5	2362.5			

Therefore, the coordinates, $\overline{X}$ and $\overline{Y}$, of the centroid of the trapezoidal region are given by the two formulas

$$\overline{X} = \frac{\sum \bar{x}A}{\sum A} = \frac{7807.5}{405} = 19.277 \text{ and } \overline{Y} = \frac{\sum \bar{y}A}{\sum A} = \frac{2812.5}{405} = 6.944$$

and the location of the centroid is indicated on the diagram above.

a) Semicircle

To calculate the centroid of the semicircle shown in figure 2, we need to calculate its area A as follows: The equation of the circle is $x^2 + y^2 = r^2$ where r is its radius. Consider a vertical strip of infinitesimally small thickness ‘dx’ at any ‘x’ having height of ‘y’. Let the area of this strip be dA = ydx. As the strip moves along the x axis from ‘0’ to ‘r’, we need to integrate the dA equation from ‘0’ to ‘r’.

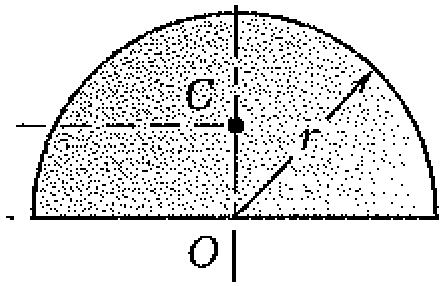

Figure2. A Semicircle

The MATLAB code for the calculation is shown in figure 3.

```
syms r theta  %defining the symbols r & theta for the integration.
x=r*cos(theta); % x in polar coordinates
y=r*sin(theta); % y in polar coordinates
dx=diff(x,theta);
dy=diff(y,theta); %taking derivative to obtain the area A of the circle
dA=x*dy;
A=int(dA,0,pi);
% To obtain the centroid of the circle Yc = (∫ydA)/A and Xc = (∫xdA1)/A1
dA1=y*dx;
A1=int(dA1,0,pi);
cgy=y*dA;
cgx=x*dA1;
Yc=int(cgy,0,pi)/A;
Xc=int(cgx,0,pi)/A1;
fprintf('The Area formed by the semicircle is: ');pretty(A)
fprintf('The X coordinate of the centroid is: ');pretty(Xc)
fprintf('The Y coordinate of the centroid is: ');pretty(Yc)
```

Figure3. MATLAB code for the Centroid of a Semicircle

The MATLAB output is shown in figure 4

```
The Area formed by the semicircle is:
                              2
                         1/2 r  pi
The X coordinate of the centroid is:
                             0
The Y coordinate of the centroid is:
                              r
                         4/3 ----
                              pi
```

Figure4. The X and Y coordinate of the Centroid of a Semicircle

For whatever numerical value of r, we can get the numerical values of the centroid.

For the quarter-circular region shown in figure 5, we simply change the limits of integration from 0 to $\pi/2$ and the result is shown in figure 6.

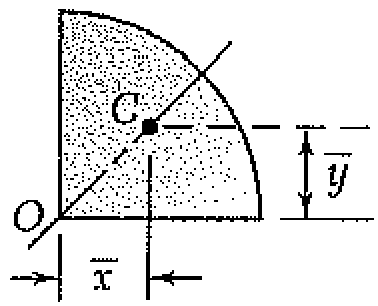

Figure5. A quarter-circular region

The Area formed by the quadrant of the circle is:

```
           2
      1/4 r  pi
```

The X coordinate of the centroid is:

```
          r
      4/3 ----
          pi
```

The Y coordinate of the centroid is:

```
          r
      4/3 ----
          pi
```

Figure6. The X and Y coordinate of the Centroid of a Quarter-circular region

b) Semielliptical

To calculate the centroid of the ellipse (very similar with the circle), we need to calculate its area A as follows: The equation of the ellipse is $(x/a)^2 + (y/b)^2 = 1$ where a and b are its x and y axes. Consider a vertical strip of infinitesimally small thickness 'dx' at any 'x' having height of 'y'. Let the area of this strip be dA = ydx. As the strip moves along the x axis from '0' to 'a', we need to integrate the dA equation from '0' to 'a'. The MATLAB code for the calculation is shown in figure 7.

```
syms a b theta % defining the symbols for integration
x=a*cos(theta);
y=b*sin(theta);% both x and y converted to polar coordinates
dx=diff(x,theta);
dy=diff(y,theta);%taking the derivatives
dA=x*dy;
% To obtain the centroid of the circle Yc = (∫ydA)/A and Xc = (∫xdA1)/A1
A=int(dA,0,pi);
dA1=y*dx;
```

```
A1=int(dA1,0,pi);
cgy=y*dA;
cgx=x*dA1;
Yc=int(cgy,0,pi)/A;
Xc=int(cgx,0,pi)/A1;
fprintf('The Area formed by the semiellipse is: ');pretty(A)
fprintf('The X coordinate of the centroid is: ');pretty(Xc)
fprintf('The Y coordinate of the centroid is: ');pretty(Yc)
```

Figure7. MATLAB code for the Centroid of a Semi-ellipse.

The MATLAB output is shown in figure 8.

```
The Area formed by the semiellipse is:
                         1/2 a b pi

The X coordinate of the centroid is:
                             0

The Y coordinate of the centroid is:
                             b
                         4/3 ----
                             pi
```

Figure8. The X and Y coordinate of the Centroid of a Semi-ellipse

For the quarter-elliptical region, we simply change the limits of integration from 0 to $\pi/2$ and the result is shown in figure 9.

```
The Area formed by the quarter-ellipse is:
                         1/4 a b pi
The X coordinate of its centroid is:
                             a
                         4/3 ----
                             pi
The Y coordinate of its centroid is:
                             b
                         4/3 ----
                             pi
```

Figure9. The X and Y coordinate of the Centroid of a Quarter-elliptical region

c) General spandrel

To calculate the centroid of the general spandrel shown in figure 10, we need to calculate its area A as follows: The equation of the general spandrel is $y = (h/a^n)\, x^n$ where a and h

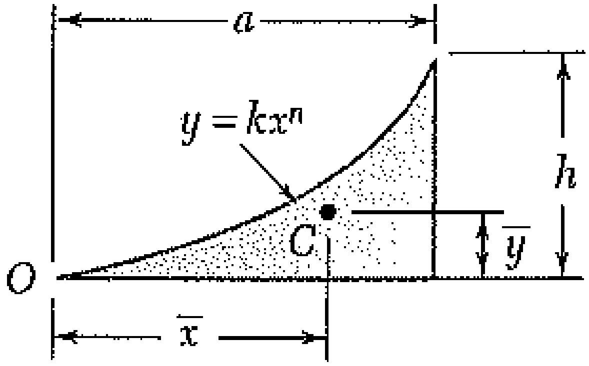

Figure10. A general spandrel region

are constants. Consider a vertical strip of infinitesimally small thickness 'dx' at any 'x' having height of 'y'. Let the area of this strip be dA = ydx. As the strip moves along the x axis from '0' to 'a', we need to integrate the dA equation from '0' to 'a'. The MATLAB code for the calculation is shown in figure 11.

```
%general spandrel
syms a h x n %symbols for integration
y=(h./(a.^n)).*x^n; % equation for the parabolic spandrel
dx=diff(x); % differentiation
dA=y*dx;
A=int(dA,0,a); %integration to find area A
cgy=x*dA;
cgx=0.5*y*dA;
Xc=int(cgy,0,a)/A;
Yc=int(cgx,0,a)/A;
fprintf('The Area formed by the parabolic spandrel is: ');pretty(A)
fprintf('The X coordinate of the centroid is: ');pretty(Xc)
fprintf('The Y coordinate of the centroid is: ');pretty(Yc)
```

Figure11. MATLAB code for the Centroid of a general spandrel region.

The MATLAB output is shown in figure 12

```
The Area formed by the parabolic spandrel is:
                               a h
                              -----
                              n + 1

The X coordinate of the centroid is:
                             a (n + 1)
                             ---------
                              n + 2
```

The Y coordinate of the centroid is:

$$\frac{1}{2}\,\frac{h\,(n+1)}{2n+1}$$

Figure12. The X and Y coordinate of the Centroid of a general spandrel region

For the case of n =2 then we have a parabolic spandrel of the form $y = (h/a^2)\,x^2$. So, the output is shown in figure 13

The Area formed by the parabolic spandrel is:

1/3 h a

The X coordinate of the centroid is:

3/4 a

The Y coordinate of the centroid is:

3/10 h

Figure13. The X and Y coordinate of the Centroid of a Parabolic spandrel region

The above ideas can be extended to the semi-parabolic and parabolic areas shown in figure 14

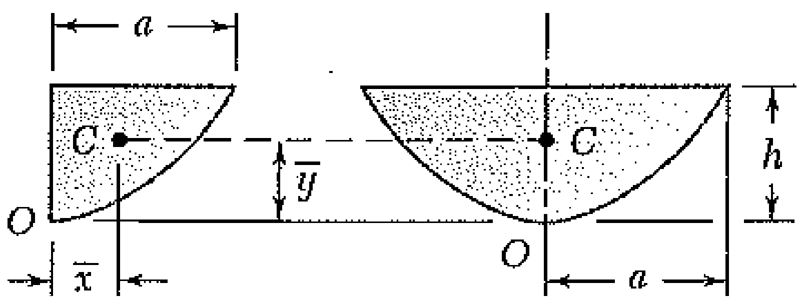

Figure14. Semi-parabolic and parabolic areas

The MATLAB output for the semi-parabolic region is shown in figure 15

The Area formed by the semi-parabolic area is:

2/3 a h

The X coordinate of the centroid is:

3/8 a

The Y coordinate of the centroid is:

3/5 h

Figure15. The X and Y coordinate of the Centroid of a semi-parabolic region

Finally, the MATALB output for the parabolic region is shown in figure 16

The Area formed by the parabolic area is:
4/3 a h
The X coordinate of the centroid is:
0
The Y coordinate of the centroid is:
3/5 h

Figure16. The X and Y coordinate of the Centroid of a parabolic region

Conclusion

This research project involved centroids as applied to the engineering course STATICS. We became familiar with their calculation by hand and using MATLAB. We presented a semicircular and quarter-circular area, semielliptical and quarter-elliptical area and a general spandrel where for n = 2 we have a parabolic spandrel as well as semi-parabolic and parabolic areas. We are very satisfied with our results. This work can be extended for three-dimensional objects such as paraboloids, ellipsoids and even hyperboloids. The reader is encouraged to become familiar with the integration calculations before using the software.

References

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