2022 HAWAII UNIVERSITY INTERNATIONAL CONFERENCES
Science, Technology & Engineering, Arts, Mathematics & Education June 7 - 9, 2022
Prince Waikiki Resort, Honolulu, Hawaii

A New Design of Premium Payments and Insurance Benefits in Various Forms of Life Insurance Designed to Protect the Insurer against Adverse Selection

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Abstract

In this paper, we discuss ways to mitigate adverse selection in life insurance with alternative policy designs, which involve different arrangements of premium payments and various benefits. We focus on three types of life insurance: term or whole life insurance, life annuity, and endowment insurance. We propose some improvements in product designs, which are expected to reduce adverse selection. Among them are annually increasing benefit for life insurance with level premium, annually decreasing benefit for life annuity paid with single premium, and endowment insurance with premium refund.

Key Words: premium payment method; adverse selection; insurance benefit; life insurance; life annuity; endowment insurance

Introduction

Adverse selection is considered to be an important issue in insurance markets. The theory of adverse selection in insurance market was introduced by Rothschild and Stiglitz (1976) and has since then been developed and extended in many ways. There exists quite extensive research literature on adverse selection, and adverse selection is of significance in business practice and public policy. Mimra and Wambach (2014) provide a survey of the viewpoints to understand existence and efficiency of equilibrium in competitive insurance market with adverse selection and find that recent results shed more light on the debate whether adverse selection provided an efficient rationale for regulating the market. Cohen and Siegelman (2010) review and evaluate the empirical literature on adverse selection in insurance market. They focus on empirical work that tests the basic coverage-risk prediction of adverse selection theory, according to which policyholders who purchase more insurance coverage tend to be riskier. They also discuss various reasons for lack of evidence of a coverage-risk in some insurance policies. However, in practice, there is an important issue in adverse selection that is rarely, if at all, discussed in the existing literature: Premium payment pattern and insurance benefits pattern. Even though insurance premium payments and benefits are at the back end of the insurance policy, a good design of payments and benefits arrangements could mitigate the consequences of missing information from the front-end of the insurance policy, i.e., from the underwriting procedures. We propose a strategy of different methods of premium payments and different arrangements for benefit payments to beneficiaries. Or analysis indicates that this new design could mitigate the undesirable effects of adverse selection.

Generally, insurance companies decrease adverse selection of the insured through fair pricing, formulating corresponding provisions of insurance contract，designing optimal (i.e., equilibrium based) insurance contracts with incentive compatibility mechanism (Whinston, 1983, Sung, 2005, Boadway, al et., Stiglitz and Yun, 2013), and limiting the behavior of risk taking from the insured. Fair pricing tends to decrease information asymmetry and improve the accuracy of forecasting and measuring risk[1].

[1]McCartly and Mitchel (2003) examine the markets of life insurance and annuity of three countries of U.S, UK and Japanese and find that in U.S. and UK, the mortality of insured with life insurance is lower than that of the population, especially in whole life insurance. This means that underwriting could reduce information asymmetry and prevent adverse selection. However, in Japan, there are opposite phenomena. Especially, there are more serious adverse selection problems in life insurance for female insured. Another finding is that adverse selection has more its effects in voluntary annuity: mortality of policy holders is worse than that of insured with whole life insurance.

However, different patterns of insurance payments and benefits arrangements can also impact on the adverse selection of the policy. For example, level premiums is likely to be more desirable for policyholders than annual renewable term increasing premium, as the former avoids the prohibitive cost in the later years of policy, due to mortality increasing with age. Additionally, policyholders may experience declining earning ability in the later years. However, the disadvantage of level premium should also be noted. Level-premium policy could result in adverse selection more than single premium, especially, when the claims happen at early policy durations.

The earlier the events, the same amount of benefits will be distributed to the beneficiary, if the proportion of the earlier events surpass the probability used in designing the policy, the payments from level-premium policy would not help cover the pooling risk as original design for this policy, that means, the adverse selection happens; however, the risk pooling is well maintained with single-premium payment policy, since all the funds are collected before the occurrences of events, the chance of mis-match of risks and pooling are less than that in level premium policy. Formally, adverse selection may manifest itself in different ways. First, probability distribution of mortality assumed in the policy design may differ from that of the insured. This may be due to the fact that people with longer life expectancy have less interest in life insurance policies with level-premium, since those type people usually are wealthy and healthy, they can afford and tend to pay the single premium, which is less than the present value of the level premiums; on the other hand, people with shorter life expectancy prefer to pay the level premium, since total amount of out-of-pocket money is less for them than that of single premium policy. These two different behaviors would self-select people into different policies, which breaks the risk-pooling of the originally designed level-premium policy. This might result in more earlier claim payments for level premium policy than that with single premium. The selective lapse is that people with different health status tends to behave differently in the same plan. Since the health status differs as time goes by, those insured with better health can seek to purchase alternative low-rate policy due to competitive nature of the markets; however, policyholders whose health deteriorates tend to keep their policies, because they can hardly find better policy. We notice that varying the benefit amount of insurance, which is seldom used in the United States, is an alternative way to address the adverse selection issue. But, as we emphasize later in this section, products of this type have appeared in China, the fast-growing China's life insurance market. This is an innovative and potentially valuable way to address the issue of adverse selection in life insurance.

Level-premium policy is a viable insurance policy. Even though single premium policy is usually offered with a discount, for example, the data from OKbao.com (OK insurance company website) shows that the average premium will be reduced by 28% if people buy single premium policies, people are still more likely to buy level premium policies due to their limited payment abilities for the long-term life insurance with great insurance benefits. In China, the sale amount of level premium policies is higher than that of single premium. In order to mitigate the potential effects from adverse selection, some insurance companies (Xu, 2014, p. 217) in China have designed a contract clause, which stipulates that the beneficiary of the insured can only get the refund of the premium paid in first year of contract term, if the death event due to disease occurs in first year. We believe that only limiting the claim in the first year after the policy issuance won't reduce adverse selection enough. Therefore, it is important to develop strategy to reduce adverse selection more effectively in life insurance products of long term with level premium. As it is indicated later, level premium policies with annually increasing benefits may be more effective to reduce the adverse selection.

Interestingly, the adverse selection in annuity markets results from longevity risk. For example, for an annuity with single premium paid at the beginning of policy issuance, the higher the longevity risk is, the more accumulated annuity benefit the annuitant is expected to receive for the same amount of premium paid, while less accumulated annuity benefit is expected to be paid for an annuitant with higher death risk. Therefore, the customers with higher death risks are less willing to buy a life annuity. Both industrial and academic experimental evidence indicate that households typically view annuity as an increase in their risk rather than a decrease (Reichling and Smetters, 2015). Hence, a greater level of risk aversion will reduce annuitization, while medical annuities (annuity with long term care insurance) could help restore more annuitization by balancing the longevity and healthcare costs risks. However, it is unclear whether consumers would be comfortable buying long term care insurance from a firm that subsequently profits from the consumers with a short life span and with which a potential (or just perceived) adverse selection problem is associated (Reichling and Smetters, 2015). Therefore, it is important to design annuity product with less risks due to annuitant's earlier death so as to increase the market demand of annuity insurance.

The main assumption of the adverse selection is information asymmetry and insured has her own risk-prediction advantage over the insurers, then the insured could purchase ill-priced insurance policy from insurer. However there are mixed evidences that the insured have such advantage, some show that insured sometimes have better knowledge of the risk of the insured; some show that the selection is multi-dimensional, in some dimension, the selection is not significant (Siegelman, 2004; McCarthy, 2010; Harris, 2016; Finkelstein, 2004). If we assume the insurer doesn’t have better risk estimation over insured themselves, insurers still use the regular mortality table, of which the mortality rate is different from that of the policy holders, to calculate the actuarially fair valuation of the products. It is still worthwhile to design life insurance products to counteract the tendency of adverse selection. Although the formulas for calculating premiums and benefits of three kinds of life insurance in this paper are based essentially on the traditional actuarial calculations with the same expected present value of total premium payments, comparisons between our design and the traditional products will shed more lights on their effectiveness against adverse selection.

In this article, we mainly discuss the design of different payment patterns for premiums and policy benefit arrangements to reduce adverse selection. We focus on three types of insurance contracts: life insurance, life annuity, and endowment life insurance. We compare the insured benefits obtained by the beneficiaries and premium paid by the insured for two life insurance products, one with level benefit and the other with annually increasing benefit. We show the problem of the adverse selection on life annuity caused by the single premium paid at the time of policy issuance, and illustrate effective ways to mitigate it. We also present an endowment insurance with adverse-selection reduction design, in which the insurer will return the single premium for survival benefit to the beneficiary if the insured dies before the end of insurance term; the insurer will also return the single premium for death benefit to the insured if he (she) survives at the end of insurance term. We show how actuarial techniques help illustrate the reasons why our product design can reduce adverse selection. We present the formulas for calculating the reserves of these insurance products. In the end, we carry out numerical analysis.

1. An annually increasing term (whole) life insurance with level premium

Level benefit-type term (whole) life insurance policy with level premium has the tendency to attract more clients with shorter life span, because the insured with earlier claim will have the same benefits as those with later claims but the out-of-pocket premium payments are lower. Even though life insurance companies will refund premium paid by the insured as an additional benefit to the insured when the claim event occurs, this incentive cannot change the fact that insured with shorter life expectancy can benefit more from level-premium policies and self-select themselves into these policies. In this paper, instead of level benefit, we propose annually increasing benefit (benefits form an increasing arithmetic sequence from issuance till the death, the difference between two adjacent benefits is constant and equals to the amount of first year benefit). The earlier the death event occurs and the less the accumulated premium the insured paid, the less the insured benefit he/she is expected to obtain. This is type of benefit design would deter people with shorter life expectancy to choose this policy and it can bring obstacles for the agent to miss-sell (offering a high premium-high commission product and then some years later trying to switch the insured into another product). To simplify our analysis, we neglect waiting period for life insurance and guaranteed period for annuity insurance.

1.1.Fully discrete level premium and insured benefit at the end of the year of death

Assume that the level premium is paid at the beginning of each year and the annually linearly increasing benefit is given to the beneficiary at the end of the year of the insured's death. In order to compare with traditional life insurance with level insured benefit to the beneficiary, we assume that the level insured benefit to the beneficiary is C, and let annually increasing insured benefit to the beneficiary be C_1 if the insured dies at the first year after the policy is issued. The insurance term is n and the age of the insured at policy issuance is x. For convenience, we assume that level premiums are the same under two different policies (pure level benefit vs the annually linearly increasing benefit to the beneficiary). Based on equivalence of present values of premiums, we have:

$$P^1_{x:\overline{n}|} = \frac{CA^1_{x:\overline{n}|}}{\ddot{a}_{x:\overline{n}|}} = \frac{C_1 \cdot \sum_{k=0}^{n-1} v^{k+1}(k+1)\,{}_k p_x q_{x+k}}{\ddot{a}_{x:\overline{n}|}} = \frac{C_1 \cdot (IA)^1_{x:\overline{n}|}}{\ddot{a}_{x:\overline{n}|}} \tag{1}$$

From equation (1), we know:

$$C_1 (IA)^1_{x:\overline{n}|} = CA^1_{x:\overline{n}|}, \tag{2}$$

and

$$C_1 = \frac{C \cdot A^{1}_{x:\overline{n}|}}{(IA)^{1}_{x:\overline{n}|}} \tag{3}$$

$(k+1)C_1 - C > 0$ is the extra benefit obtained by the insured's beneficiary at the end of the year of death, i.e., at time $k+1$, $k = 0,1,2,...,n-1$, between the payment methods of annually increasing benefit and level benefit. The extra benefit is positive if the claim event occurs.

$$(k+1)C_1 - C = (k+1)\frac{C \cdot A^{1}_{x:\overline{n}|}}{(IA)^{1}_{x:\overline{n}|}} - C\,. \tag{4}$$

This extra benefit can be less than that from level benefit plan if the claim occurs earlier in the policy term; this extra benefit is greater than that with level benefit, if the claim event occurs at the end of year of $k+1$ satisfying inequality (5):

$$k+1 > \frac{(IA)^{1}_{x:\overline{n}|}}{A^{1}_{x:\overline{n}|}}, k = 0,1,2,....n-1 \tag{5}$$

If the claim event occurs at the end of insurance term, the benefit with annually increasing benefit is

$$nC_1 = \frac{C \cdot nA^{1}_{x:\overline{n}|}}{(IA)^{1}_{x:\overline{n}|}}, \tag{6}$$

and extra benefit is

$$nC_1 - C = C \cdot \frac{nA^{1}_{x:\overline{n}|} - (IA)^{1}_{x:\overline{n}|}}{(IA)^{1}_{x:\overline{n}|}}\quad . \tag{7}$$

Since $nA^{1}_{x:\overline{n}|} - (IA)^{1}_{x:\overline{n}|} > 0$, the benefit obtained by the insured's beneficiary when the claim event occurs at the end of insurance term is larger than C. However, if the claim event occurs at the end of first year of policy issuance, the extra benefit is as in equation (8):

$$C_1 - C = C \cdot \frac{A^1_{x:\overline{n}|} - (IA)^1_{x:\overline{n}|}}{(IA)^1_{x:\overline{n}|}} \qquad (8)$$

Since $A^1_{x:\overline{n}|} - (IA)^1_{x:\overline{n}|} < 0$, the benefit obtained by the insured's beneficiary is much less than C. Based on the analysis of extra benefit for claims of different time of the term, there is less incentive for the insured to take the advantage of the policy in regards to earlier claim, the risk of adverse selection under annually increasing benefit life insurance is expected to be smaller.

For whole life insurance, let C_{1x} be the death benefit in the first year (assuming a policy with a variable benefit). Then we have,

$$C_{1x} = \frac{C \cdot A_x}{(IA)_x}. \qquad (9)$$

Based on the equation (9), similarly to the discussion in the discrete case, we have the same conclusion that the risk of adverse selection is expected to be reduced.

1.2. Fully discrete level premium and insured benefit paid at the moment of death

Similarly, we can calculate the insured benefit obtained by the beneficiary if the claim occurs at any moment of k th year after the policy issuance, that is:

$$(k+1)\overline{C}_1 = \frac{(k+1)C \cdot \overline{A}^1_{x:\overline{n}|}}{(I\overline{A})^1_{x:\overline{n}|}}, k = 0,1,2,...,n-1, \qquad (10)$$

and for whole life insurance, let $\overline{C}_{1x}$ be the benefit given to the insured's beneficiary at the moment of he (she) dies during the first policy year， we have:

$$(k+1)\overline{C}_{1x} = \frac{(k+1)C \cdot \overline{A}_x}{(I\overline{A})_x}, k = 0,1,2,...,n-1 \qquad (11)$$

If the distribution of deaths is uniform within the year of death (i.e., under the uniform distribution of deaths, or UDD assumption), we have

$$\bar{A}^{1}_{x:\overline{n}|} = \frac{i}{\delta} A^{1}_{x:\overline{n}|}, \tag{12}$$

$$\bar{A}_x = \frac{i}{\delta} A_x, \tag{13}$$

$$(I\bar{A})^{1}_{x:\overline{n}|} \approx \frac{i}{\delta} (IA)^{1}_{x:\overline{n}|} \ , \tag{14}$$

and

$$(I\bar{A})_x \approx \frac{i}{\delta} (IA)_x \ , \tag{15}$$

Putting equations of (12) and (14) into equation (10) and putting equations of (13) and (15) into equation (11), we have, under the UDD assumption,

$$C_1 = \bar{C}_1 \tag{16}$$

and

$$C_{1x} = \bar{C}_{1x}. \tag{17}$$

As the above equations show, based on the same reasoning, we have that the risk of adverse selection is expected to be reduced through the annually increasing benefit plan.

2. Level annuity with annually decreasing death benefit and immediate single premium

2.1. Temporary life annuity with term n for $n < ¥$

There is a problem of adverse selection for level life annuities, too. It is difficult for the policy issuers to price the annuity properly and achieve the risk pooling of different risk types. Because healthy people with long life expectancy are willing to purchase level life annuity; however, those unhealthy people with higher mortality (low longevity risk) are less likely to purchase it due to less annuity benefit. As indicated by Heijdra and Reijnders (2012), low mortality (high-risk from the perspective of insurance company) individuals are over-represented in the annuity market. This results in higher prices (or lower return) and makes the policy even less attractive for less healthy individuals.

Traditionally, insurance companies issue the life annuity with guaranteed death benefit to attract unhealthy people. In this section, we propose an alternative. We present a term life annuity insurance, which pays level annuity to the annuitants (x) who survive and will be paid decreasing benefit at their death. We will show in the following sections that this type of annuity product can mitigate longevity-improvement risk, which is important to insurers, especially in current time when longevity improvement risk faced by the insurers has increased, with the progress of biotechnology, increasing standards of living, and improvements in public health.

Let B be the level annuity annual benefit for traditional policy and n be the insurance term. Assume the annuity benefit includes two parts. One part is level annuity benefit, which will be paid to the insured while they survive, and the other part is annually decreasing death benefit. Let B_0 be the level annuity benefit and $(n-k)B_0$ ($k = 0, 1, 2, ..., n\text{-}1$) be the benefit returned to the beneficiary if the insured's death occurs in the k-th year. We assume that the annuity is paid for with a single premium at the beginning of the policy, the level annuity is given at the end of each year if the insured survives, and annually decreasing benefit is given to the beneficiary at the end of the year of the insured's death. For the convenience of comparison with traditional annuity, we let the total pure benefit paid by level annuity with annually decreasing death benefit be equal to that paid by traditional annuity, that is,

$$B \cdot a_{x:\overline{n}|} = B \cdot \sum_{k=1}^{n-1} v^k \, {}_k p_x = B_0 \sum_{k=1}^{n-1} v^k \, {}_k p_x + B_0 \sum_{k=0}^{n-1} (n-k) v^{k+1} \, {}_k p_x q_{x+k+1} = B_0 a_{x:\overline{n}|} + B_0 (DA)^1_{x:\overline{n}|}, \tag{19}$$

Then we get

$$B_0 = \frac{B \cdot a_{x:\overline{n}|}}{a_{x:\overline{n}|} + (DA)^1_{x:\overline{n}|}}. \tag{20}$$

From equation (20) we see that the benefit obtained by the death beneficiary is much greater than B if the insured dies during the first year of the annuity. Equation (21) gives the cumulative present value of benefit obtained by the annuitant and his (her) beneficiary if the death occurs at the k-th year,

$$\begin{aligned} B_{1k+1} &= B_0 a_{x:\overline{k}|} + (n-k)B_0 v^{k+1} \\ &= \frac{B \cdot a_{x:\overline{n}|}}{a_{x:\overline{n}|} + (DA)^1_{x:\overline{n}|}} \left(a_{x:\overline{k}|} + (n-k)v^{k+1} \right), k = 0,1,2,\ldots,n-1 \end{aligned} \tag{21}$$

and equation (22) gives the benefit difference ($B1$) between traditional annuity insurance and annuity insurance with annually decreasing death benefit if death event occurs at time $k+1$:

$$\begin{aligned} B_{3k+1} &= B_{1k+1} - Ba_{\overline{k+1}|} \\ &= B_0 a_{\overline{k}|} + (n-k)B_0 v^{k+1} - Ba_{\overline{k+1}|} \\ &= \frac{B \cdot a_{x:\overline{n}|}}{a_{x:\overline{n}|} + (DA)^1_{x:\overline{n}|}} \left(a_{\overline{k}|} + (n-k)v^{k+1} \right) - Ba_{\overline{k+1}|}, k = 0,1,2,n-1 \end{aligned} \tag{22}$$

And equation (23) shows the difference of the present value between these two types of annuity insurance at time n,

$$B_{1n} = \left(B_0 - B \right) a_{\overline{n}|} \tag{23}$$

We know from equation (20), since $B_0 < B$, $B_{1n} < 0$.

Derived from equation (22) we see that when

$$k < n + \frac{a_{x:\overline{k-1}|}}{v^{k+1}} - a_{\overline{k}|} \cdot \frac{a_{x:\overline{n}|} + (DA)^1_{x:\overline{n}|}}{a_{x:\overline{n}|} v^{k+1}}, \tag{24}$$

the cumulative present value of survival annuity and decreasing death benefit at time

k is greater than that of the traditional annuity insurance. This strategy brings less healthy people (with lower longevity risk) more incentives to purchase such a plan. This design hedges the risk of adverse selection in this type of policies.

2.2. Whole life annuity

We assume that the term of annually decreasing death benefit is m, and the annually decreasing death benefit, $(m-k)B_{0x}, k=0,1,2,...,m-1,$ will be given at the end of the k th year when the insured dies at any moment during that year. Otherwise, the level annuity B_{0x} will be paid at the end of k th year. And during the period of $m+1$ to infinity, the insured obtains annuity benefit, B_{0x}, if he (she) is alive at the time $m+1+j, j=0,1,2,....$ .Then,

$$\begin{aligned} B\cdot a_x &= B\cdot\sum_{k=1}^{\infty} v^k\,{}_k p_x = B_{0x}\sum_{k=1}^{\infty} v^k\,{}_k p_x + B_{0x}\sum_{k=0}^{m-1}(m-k)v^{k+1}\,{}_k p_x q_{x+k+1} \\ &= B_0 a_x + B_{0x}(DA)^1_{x:\overline{m}|} \end{aligned} \tag{25}$$

From equation (26), we obtain:

$$B_{0x} = \frac{B\cdot a_x}{a_x + (DA)^1_{x:\overline{m}|}}, \tag{26}$$

If the death benefit is given at the moment of the death and the death event follows the uniform distribution, the death benefit given at the k th year of insurance term is:

$$(n-k)\overline{B}_{0x} = \frac{(n-k)B\cdot a_{x:}}{a_x + (D\overline{A})^1_{x:\overline{m}|}} \approx \frac{(n-k)B\cdot a_x}{a_x + \frac{i}{\delta}(DA)^1_{x:\overline{m}|}}. \tag{27}$$

Since a level annuity, B_{0x}, means for annuitants with earlier death less accumulated, from the insurers' standpoint, high-risk annuitants are those who are likely to live longer than what their observable attributes, such as age, would otherwise suggest (Finkelstein and James Poterba, 2004). Therefore, it is quite clear from the analysis above that adverse selection due to the increase in longevity risk from very healthy lives is reduced by limiting the tail of annuity payment or a death benefit.

3. Endowment insurance with different death benefit and survival benefit

Due to the even higher premium in endowment insurance with level premium or single premium, it is harder to attract people of low-risk to achieve the risk-pooling. People who purchase endowment insurance tends to be those who lives longer than what insurers has expected; those whose expected life is shorter would more likely to purchase term life insurance. In this section, we design two new endowment insurance plans which can mitigate adverse selection.

3.1. Endowment insurance with level premium

We Assume that insured benefit with level premium for traditional endowment insurance is D, insured benefit for new endowment insurance is D_1, and the insurer will return the annual pure premium for survival benefit to the beneficiary if the insured dies before insurance term and the insurer will also return the annual premium for death benefit to the insured if he(she) survives at the end of insurance term. To compare with traditional endowment insurance, we let the total pure premium paid by new endowment insurance be equal to that paid by traditional endowment insurance, that is,

$$DP_{x:\overline{n}|}\ddot{a}_{x:\overline{n}|} = D_1\left(\sum_{k=0}^{n-1}\left(v^{k+1} + P_{x:\overline{k}|}^{\ \ 1}\ddot{a}_{x:\overline{k}|}\right){}_k p_x q_{x+k+1}\right) + D_1\left(v^n + P^1_{x:\overline{n}|}\ddot{a}_{x:\overline{n}|}\right){}_n p_x \quad (28)$$

and

$$D_1 = \frac{DP_{x:\overline{n}|}\ddot{a}_{x:\overline{n}|}}{\left(\sum_{k=0}^{n-1}\left(v^{k+1} + P_{x:\overline{k}|}^{\ 1}\ddot{a}_{x:\overline{k}|}\right){}_k p_x q_{x+k+1}\right) + \left(v^n + P_{x:\overline{n}|}^{1}\ddot{a}_{x:\overline{n}|}\right){}_n p_x} \tag{29}$$

If the insured dies during k th year, $k = 1, 2, \ldots n$, his/her beneficiary will get the benefit of the sum of D_1 and the future value of the refund of the pure premium he/she has paid for his/her survival benefit, $P_{x:\overline{n}|}^{\ 1}\ddot{a}_{\overline{k}|} \cdot D_1 / v^k, k = 1, 2, \ldots n$, at the end of k th year. Otherwise, the insured will get the benefit of the sum of D_1 and the future value of the refund of the pure premium he/she has paid for his/her death benefit, $P_{x:\overline{n}|}^{1}\ddot{a}_{\overline{n}|} \cdot D_1 / v^n$, at the end of insurance term.

3.2. Endowment insurance with immediate single pure premium

Assume that the benefit with immediate single premium is G for traditional endowment insurance, and insured benefit given by the insurer is G_1 for new endowment insurance. In our proposal, the insurer will return the single pure premium for survival benefit to the beneficiary if the insured dies before the end of insurance term and the insurer will also return the single pure premium for death benefit to the insured if he(she) survives at the end of insurance term. For the comparison with traditional endowment insurance, we let the total pure premium paid for new endowment insurance be equal to that paid by traditional endowment insurance, that is,

$$GA_{x:\overline{n}|} = G_1\left(\sum_{k=0}^{n-1}\left(v^{k+1} + A_{x:\overline{n}|}^{\ 1}\right){}_k p_x q_{x+k+1}\right) + G_1\left(v^n + A_{x:\overline{n}|}^{1}\right){}_n p_x, \tag{30}$$

and

$$G_1 = \frac{GA_{x:\overline{n}|}}{\left(\sum_{k=0}^{n-1}\left(v^{k+1} + A_{x:\overline{n}|}^{\;1}\right)\,_k p_x q_{x+k+1}\right) + \left(v^n + A_{x:\overline{n}|}^{1}\right)\,_n p_x} \tag{31}$$

If the insured dies during k th year, $k = 1, 2, \ldots n$, his/her beneficiary will get the benefit of the sum of G_1 and the future value of pure premium he/she has paid for his/her survival benefit, $A_{x:\overline{n}|}^{\;1} \cdot G_1 / v^k, k = 1, 2, \ldots n$, at the end of k th year. Otherwise, the insured will get the benefit of the sum of G_1 and the future value of pure premium he/she has paid for his/her death benefit, $A_{x:\overline{n}|}^{1} \cdot G_1 / v^n$, at the end of insurance term.

We make comparisons through numerical analysis in next section (Please see Table 4 and 5), since it impossible to compare the insured benefit at different time (especially at earlier stage of contract term) symbolically.

Table 1 lists the formulas for calculation of reserves of three insurance products: term or whole life insurance with annually increasing benefit and level premium payment; annuity with decreasing death benefit and single premium, and endowment life insurance with different value of death and survival benefit. We call them as A, B and C respectively.

Table 1. Formulas for calculating the reserves of products: Aa, Bb and Cc

Products	The formulas for calculation of reserves
Aa	${}_kV_{x:\overline{n}|}^{1} = \left(IA\right)_{x+k:\overline{n-k}|}^{1} - P_{x:\overline{n}|}^{1}\,\ddot{a}_{x+k:\overline{n-k}|}$, (32) Term life insurance ${}_kV_x = \left(IA\right)_{x+k:\overline{n-k}|}^{1} - P_x\,\ddot{a}_{x+k:\overline{n-k}|}$ (33) Whole life insurance
Bb	${}_kV(a_{x:\overline{n}|}) = a_{x:\overline{n}|} - \left(A_0\left(DA\right)_{x:\overline{k}|}^{1} + A_0 a_{x:\overline{k}|}\right)$ (34) Temporary annuity ${}_kV(a_x) = a_x - \left(A_0\left(DA\right)_{x:\overline{k}|}^{1} + A_0 a_{x:\overline{k}|}\right)$ (35) Whole life annuity

	A_0 is determined by (20) where A is set to be 1.
Cc	For endowment insurance with level premium: $_kV_{x:\overline{n}\rvert} = D_1\left(\sum_{j=0}^{k-1}\left(v^{j+1} + P_{x:\overline{j}\rvert}^{1}\ddot{a}_{x:\overline{j}\rvert}\right)\,_j p_x q_{x+j+1}\right) - P_{x:\overline{n}\rvert}\ddot{a}_{x+k:\overline{n-k}\rvert} \qquad k < n$ $_kV_{x:\overline{n}\rvert} = 1 \qquad k = n$ (36) For endowment insurance with single premium paid at the moment of policy issuance: $_kV_{x:\overline{n}\rvert} = A_{x:\overline{n}\rvert} - G_1\left(\sum_{j=0}^{k-1}\left(v^{j+1} + A_{x:\overline{n}\rvert}^{1}\right)\,_j p_x q_{x+j+1}\right) \qquad k < n$ $_kV_{x:\overline{n}\rvert} = 1 \qquad k = n$ (37) D_1 and G_1 are determined by equations (29) and (31), where D and G are set to be 1.

4. Numerical Analysis

We assume that the risk-free interest rate $r = 0.03$, and we use the mortality table of US. 2000. Table 2 through Table 5 lists the premium rate and insured benefit of term life insurance with annually increasing death benefit and level premium, temporary annuity insurance with annually decreasing death benefit and single premium paid at the time of policy issuance and term endowment insurance with refund cash value. We assume that the insurance term n is 10 years and the insured's age x at the time of issuance is 25, 35, 45, and 55 respectively for all three types of life insurance.

Table 2. The premium rate and insurance benefit of term life insurance with annually increasing death benefit and level premium

1. $x=25, n=10$					
$P^1_{25:\overline{10}\vert}$	0.00091321				
$C_1(C=1)$	0.1761				
k	1	2	3	4	5
$C_2(k)$	0.1761	0.3522	0.5283	0.7044	0.8805
k	6	7	8	9	10
$C_2(k)$	1.0566	1.2327	1.4088	1.5849	1.7610
$x=35, n=10$					
$P^1_{35:\overline{10}\vert}$	0.0014				
$C_1(C=1)$	0.1789				
k	1	2	3	4	5
$C_2(k)$	0.1789	0.3577	0.5366	0.7155	0.8943
k	6	7	8	9	10
$C_2(k)$	1.0732	1.2621	1.4309	1.6098	1.7887
$x=45, n=10$					
$P^1_{45:\overline{10}\vert}$	0.0025				
$C_1(C=1)$	0.1677				
k	1	2	3	4	5
$C_2(k)$	0.1677	0.3355	0.5032	0.6710	0.8387
k	6	7	8	9	10
$C_2(k)$	1.0064	1.1742	1.3419	1.5097	1.6774
$x=55, n=10$					
$P^1_{55:\overline{10}\vert}$	0.0066				
$C_1(C=1)$	0.1662				
k	1	2	3	4	5
$C_2(k)$	0.1662	0.3323	0.4985	0.6646	0.8308
k	6	7	8	9	10
$C_2(k)$	0.9969	1.1631	1.3293	1.4954	1.6616

Note that $C_2(k)$ is the ratio of insurance benefit of the new life insurance to traditional life insurance with level premium for the beneficiary if the insured dies at time $k, k=1,2,\ldots 10$. We assume that the present value of premium rate is the same for these two types of term life insurance. C_1 is the insurance benefit of first year after policy issuance where C is the insurance benefit for traditional term life insurance with level premium and $C=1$.

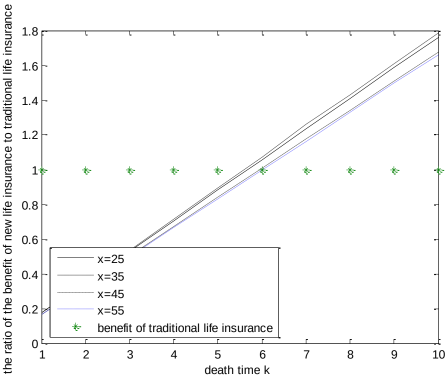

Figure 1. Comparison of the death benefits between new and traditional life insurance

Table 3. Premium rate and insurance benefit of annuity insurance with annually decreasing death benefit and immediate single premium

$x=25, n=10$					
$a_{25:\overline{10}\rvert}=8.24438$, $B_0(B=1)=0.9949, B_2=8.4864, B_4=-0.0438$					
k	1	2	3	4	5
$B_1(k)$	9.6589	9.3776	9.1317	8.9196	8.7394
$B_3(k)$	8.6880	7.4641	6.3031	5.2025	4.1597
k	6	7	8	9	10
$B_1(k)$	8.5894	8.4681	8.3738	8.3052	8.2608
$B_3(k)$	3.1722	2.2378	1.3541	0.5191	-0.2694
$x=35, n=10$					
$a_{35:\overline{10}\rvert}=8.2235$, $B_0(B=1)=0.9921, B_2=8.4630, B_4=-0.0672$					
k	1	2	3	4	5

$B_1(k)$	9.6322	9.3517	9.1065	8.8950	8.7153
$B_3(k)$	8.6614	7.4382	6.2779	5.1779	4.1356
k	6	7	8	9	10
$B_1(k)$	8.5657	8.4447	8.3507	8.2823	8.2380
$B_3(k)$	3.1485	2.2144	1.3310	0.4962	-0.2922
$x = 45, n = 10$					
$a_{45:\overline{10}|} = 8.1821$， $B_0(B=1) = 0.9866, B_2 = 8.4156, B_4 = -0.1145$					
k	1	2	3	4	5
$B_1(k)$	9.5784	9.2994	9.0556	8.8453	8.6665
$B_3(k)$	8.6075	7.3859	6.2270	5.1282	4.0868
k	6	7	8	9	10
$B_1(k)$	8.5178	8.3975	8.3040	8.2360	8.1919
$B_3(k)$	3.1006	2.1672	1.2843	0.4499	-0.3383
$x = 55, n = 10$					
$a_{55:\overline{10}|} = 8.0298$， $B_0(B=1) = 0.9662, B_2 = 8.2418, B_4 = -0.02887$					
k	1	2	3	4	5
$B_1(k)$	9.3801	9.1069	8.8782	8.6622	8.4872
$B_3(k)$	8.4093	7.1934	6.0396	4.9451	3.9074
k	6	7	8	9	10
$B_1(k)$	8.3415	8.2237	8.1322	8.0655	8.0224
$B_3(k)$	2.9243	1.9934	1.1125	0.2794	-0.5078

Note that $B_1(k)$ is the sum of cumulative present value of annuity and annually decreasing death benefit of new annuity insurance for the insured's beneficiary, $k, k = 1, 2, ..., 10$, if the insured dies at time $k, k = 1, 2, ...10$. We assume that the present value of premium rate is the same for these two annuity insurances i.e., $B = 1$. B_2 is the cumulative present value of annuity benefit of the new annuity insurance if the insured is alive at the end of insurance term. B_0 is the annuity ratio for the new annuity insurance to traditional annuity insurance because in our assumption $B = 1$. $B_3(k)$ is the difference of cumulative present value of insurance benefit between the new and traditional annuity insurance at time $k, k = 1, 2, ...n$ when $B = 1$. B_4 is the difference of cumulative present value of annuity benefit between the new annuity insurance and traditional annuity insurance if the insured survives at the end of insurance term.

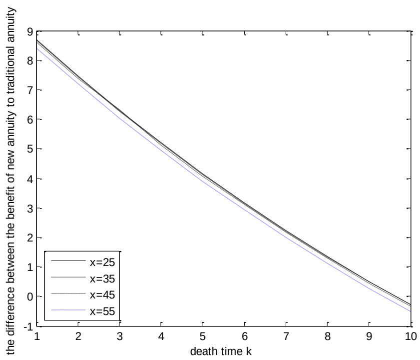

Figure 2. The difference of the claimed benefit between new and traditional annuity insurance

Table 4. Premium rate and insurance benefit of endowment insurance with refund of premium for insured death benefit or survival benefit and level premium

$x = 25, n = 10$					
$P_{25:\overline{10}\rvert} = 0.0851, \ D_1(D=1) = 0.9849, D_3 = 0.9954$					
k	1	2	3	4	5
k	6	7	8	9	10
D_2	1.5372	1.6392	1.7442	1.8524	1.9638
$x = 35, n = 10$					

$P_{35:\overline{10}\rceil} = 0.0853$, $D_1(D=1) = 0.9775, D_3 = 0.9934$					
k	1	2	3	4	5
$D_2(k)$	1.0620	1.1490	1.2387	1.3311	1.4262
k	6	7	8	9	10
$D_2(k)$	1.5242	1.6251	1.7291	1.8361	1.9464
$x = 45, n = 10$					
$P_{45:\overline{10}\rceil} = 0.0858$, $D_1(D=1) = 0.9593, D_3 = 0.9880$					
k	1	2	3	4	5
$D_2(k)$	1.0415	1.1262	1.2135	1.3033	1.3959
k	6	7	8	9	10
$D_2(k)$	1.4913	1.5895	1.6906	1.7948	1.9021
$x = 55, n = 10$					
$P_{55:\overline{10}\rceil} = 0.0875$, $D_1(D=1) = 0.9068, D_3 = 0.9773$					
k	1	2	3	4	5
$D_2(k)$	0.9823	1.0602	1.1403	1.2228	1.3079
k	6	7	8	9	10
$D_2(k)$	1.3954	1.4857	1.5786	1.6743	1.7728

Note that $D_2(k)$ is the ratio of insurance benefit of the new endowment insurance to traditional endowment insurance for the insured's beneficiary including the future value of the cumulative premium paid by the insured for the survival benefit from time 1 to $k, k = 1, 2, ..., 10$, if he/she dies at time $k, k = 1, 2, ...10$. We assume that the present value of premium rate is the same for this two endowment insurances. D_3 is the ratio of insurance benefit of the new endowment insurance to traditional endowment insurance for the insured including the future value of premium paid by the insured for the death benefit if he/she survives at the end of insurance term. D_1 is the insurance benefit excluding the premium returning to the insured or the insured's beneficiary for new endowment insurance. D is the insurance benefit for traditional endowment insurance.

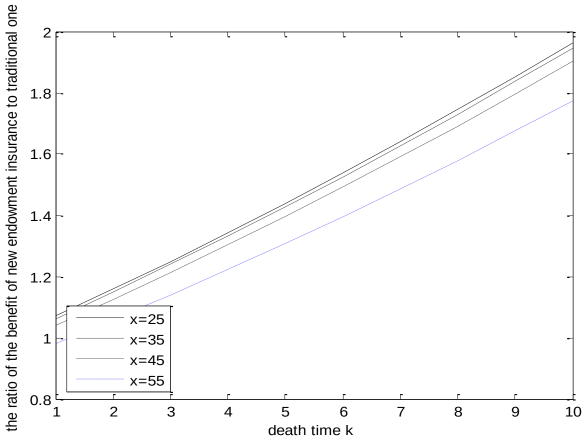

Figure 3. The claimed benefit for new endowment life with level premium and with different age at the time of policy issuance

Table 5. Premium rate and insurance benefit of endowment insurance with refund premium for insured death benefit or survival benefit and immediate single premium

$x=25, n=10$					
$A_{25:\overline{10}\rceil}$	0.7444				
$G_1(G=1)$	0.9804				
G_3	0.9909				
k	1	2	3	4	5
$G_2(k)$	1.7241	1.7464	1.7694	1.7930	1.8174
k	6	7	8	9	10
$G_2(k)$	1.8425	1.8684	1.8950	1.9225	1.9507
$x=35, n=10$					

$A_{35:\overline{10}\rceil}$	0.7447				
$G_1(G=1)$	0.9709				
G_3	0.9866				
k	1	2	3	4	5
$G_2(k)$	1.7035	1.7255	1.7481	1.7714	1.7954
k	6	7	8	9	10
$G_2(k)$	1.8202	1.8456	1.8719	1.8989	1.9268
$x=45, n=10$					
$A_{45:\overline{10}\rceil}$	0.7452				
$G_1(G=1)$	0.9484				
G_3	0.9765				
k	1	2	3	4	5
$G_2(k)$	1.6548	1.6760	1.6979	1.7203	1.7435
k	6	7	8	9	10
$G_2(k)$	1.7674	1.7919	1.8172	1.8433	1.8701
$x=55, n=10$					
$A_{55:\overline{10}\rceil}$	0.7472				
$G_1(G=1)$	0.8832				
G_3	0.9499				
k	1	2	3	4	5
$G_2(k)$	1.5118	1.5306	1.5501	1.5701	1.5907
k	6	7	8	9	10

$G_2(k)$	1.6119	1.6337	1.6563	1.6795	1.7033

Note: $G_2(k)$ is the ratio of insurance benefit of the new endowment insurance to traditional endowment insurance for the insured's beneficiary including the future value of the cumulative premium paid by the insured for the survival benefit from time 1 to $k, k=1,2,...,10$, if he/she dies at time $k, k=1,2,...10$. We assume that the present value of premium rate is the same for this two endowment insurances. G_3 is the ratio of insurance benefit of the new endowment insurance to traditional endowment insurance for the insured including the future value of premium paid by the insured for the death benefit if he/she survives at the end of insurance term. G_1 is the insurance benefit excluding the premium returning to the insured or the insured's beneficiary for the new endowment insurance. G is the insurance benefit for traditional endowment insurance.

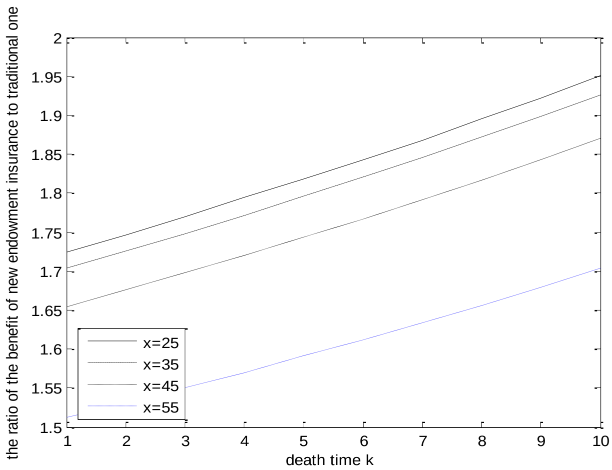

Figure 4. The claimed benefit for the new endowment life with immediate single premium and with different age at the time of policy issuance

Table 2 and Figure 1 indicate that the death benefit is annually increasing and it is less for the new term life insurance than that for traditional life insurance with level premium when $k \le 6(7)$, which means that for the new term life insurance we proposed, the earlier the death event occurs, the less benefit the insured 's beneficiary will obtain.

It also means that the more cumulative premium the insured paid, the more death benefit his/her beneficiary will obtain. Through this way, the policy with level premium can attract people with shorter expected life.

We find from Table 3 and Figure 2 that the cumulative present value of the insured benefit including annually decreasing death benefit and the survival annuity of the new annuity insurance is greater than that of pure survival benefits of traditional annuity if death occurs at time $k, k = 1, 2, ...n$ except the last year of insurance term, in which the benefit difference between the new and traditional annuity insurance is negative no matter whether the insured is survived or dead at this year (Please see the values of B_4 and $B_3(10)$ in Table 3). And the difference of insured benefit between the new and traditional annuity insurance is decreasing with the increase of the insured age at policy issuance. Therefore, the annuity insurance with annually decreasing death benefit can help decrease the adverse selection, especially it can attract unhealthy people to purchase this annuity insurance so as to hedge the longevity risk of life annuity. Moreover, since the insured benefit decreases with the increase of the age at which the insured dies, it will also help to hedge the risk of mortality improvement.

Even though people with shorter life expectancy are attracted to this new policy and they will obtain higher benefits for earlier death, the chance of self-selection of people of this type into this policy is low due to much higher premium than that of life insurance for the same insured benefit. Therefore, people, who want to hedge death risk, is more likely to buy life insurance rather than this annuity. Of course, we do not want to deny the fact that the people buying this type of annuity are expected to have shorter life than those buying pure annuity. However, it can increase risk-pooling of that of pure annuity. In addition, this new type of annuity can motivate the customers with higher risk aversion to buy annuity insurance so as to expend the limited annuity markets.

We find from Table 4 and Figure 3 that compared with the traditional endowment insurance, the death benefit for our new endowment insurance is greater except that of first year after policy issuance when the insured age is 55 at that time, and the later the insured dies, the more the death benefit his/her beneficiary will get. It is reasonable because the insured will pay more cumulative premium when he/she dies in the later time of insurance term since the premium is paid equally each year. Through the refund of the premiums for the survival benefit paid by the insured to his/her beneficiary, it will greatly decrease the adverse selection of the insured, that is, less healthy people do not like to buy endowment insurance. We also find from Table 4 that differently from traditional endowment insurance, the new endowment insurance will pay less benefit to the insured when the age x of insured at the time of policy issuance increases. It is reasonable that higher age meaning lower probability of survival. However, similarly to traditional endowment insurances，the pure premium of the new endowment insurance will increase with the increase of age because mortality is worsening morer than the improvement of survival between the age of 25 and 55.

Table 5 and Figure 4 indicate that the death benefit including the refund premium for the insured's survival benefit to his/her beneficiary is much greater especially if the death event occurs at the initial several years after policy is issued, since all premiums are paid at the time of policy issuance and much less cumulative annuity benefit is obtained by the insured due to his/her earlier death.

Conclusions

In this paper, we discuss three new types of insurance products: life insurance with annually increasing insured benefit and level premium, level annuity with annually decreasing death benefit and immediate single premium, and endowment insurance with refund premium for the insured's death; survival benefit and level premium and immediate single premium. We establish models to calculate the pure premium and insured benefit for the three insurance products and show numerically how these products can help decrease adverse. Through the numerical analyses, we can see that these new designs of products can increase the risk-pooling and help mitigate adverse selection. Our future study will focus on the empirical examination and comparison between the effect of temporary annuity (or term life insurance) and whole life annuity (or whole life insurance) on adverse selection.

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